# University of California, Santa Cruz Board of Studies in Computer Engineering 

CMPE-242:
Applied Feedback Control


HOMEWORK \#9
Due 12-Mar-2013

1. Put the following transfer functions into state space using the controller canonical form (use MATLAB to check your answers after you have done them by hand):
a. $\quad G(s)=\frac{2 s+1}{s^{2}+3 s+2}$
b. $\quad G(s)=\frac{s+3}{s\left(s^{2}+2 s+2\right)}$
c. $G(s)=\frac{3 s+6}{2 s+10} \quad$ (hint: this one has a non-zero $\boldsymbol{D}$ term, use long division first)
2. Given the following system, with zero initial conditions, find the steady state value of $\boldsymbol{x}$ to a step input:

$$
\dot{\mathbb{x}}=\left[\begin{array}{cc}
-8 & 2 \\
-4 & -2
\end{array}\right] \mathbb{x}+\left[\begin{array}{l}
0 \\
1
\end{array}\right] \text { un }
$$

3. Write down the state equations using the states indicated in the drawing below, then derive the transfer function both using block diagram manipulation and linear algebra (and check that you get the same result):

4. Consider the following two mass problem that we have been looking at in class. Let the mass of the first mass be double the second mass: $m_{1}=2 \mathrm{Kg}, \mathrm{m}_{2}=1 \mathrm{Kg}, \mathrm{k}_{1}=0.5 \mathrm{~N} / \mathrm{m}$ and $\mathrm{b}_{1}=0.05 \mathrm{Ns} / \mathrm{m}$ :

a. Derive the state space equations using the state vector: $\mathbb{X}=\left[\begin{array}{llll}x_{1} & \dot{x_{1}} & x_{2} & \dot{x_{2}}\end{array}\right]^{T}$
b. Find a similarity transform that will convert your answer in part (a) into a new set of equations with the state vector: $\mathbb{X}_{\text {new }}=\left[\begin{array}{llll}x_{1} & \dot{x_{1}} & x_{C M} & x_{C M}\end{array}\right]^{T}$, where $\mathrm{x}_{\mathrm{cm}}$ refers to the center of mass of the two masses (remember to account for the different masses of $m_{1}$ and $\mathrm{m}_{2}$ ).
i. What is the similarity transform: T and/or $\mathrm{T}^{-1}$
ii. What are the new state equations?
c. Use MATLAB to plot the step response of the equations in both part (a) and (b), with the output being: $\mathbb{y}=\left[\begin{array}{lll}x_{1} & x_{c m} & F\end{array}\right]^{T}$
i. What are the two different $\boldsymbol{C}$ and $\boldsymbol{D}$ matrices?
ii. Verify that both versions give the same response.
d. Use ACKER.M and/or PLACE.M to design a full state feedback control law that will place the closed loop poles at sdes $=-1+-j,-2+-2 j$. Use the equations developed in part 4(b).
i. Derive the Controllability Matrix
ii. Verify both the rank and the condition number of the Controllability Matrix.
iii. What are your gains, K?
iv. Plot a step response of the system with the same output as in part (c).
