UNIVERSITY OF CALIFORNIA, SANTA CRUZ
BOARD OF STUDIES IN COMPUTER ENGINEERING

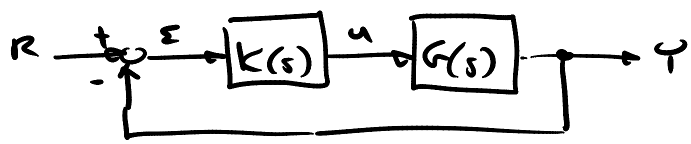
CMPE-242:
APPLIED FEEDBACK CONTROL

HOMEWORK #5
DUE 12-FEB-2013

---

1. *Digital and Continuous Equivalents*: Consider the following simple oscillatory system, with two poles on the jω axis:

$$G(s) = \frac{3}{s^2 + 3}$$



a. Design a compensator, $K_1(s)$, that will place the closed roots of this system at the desired poles of s_{des} = -2±4j. Use a unity gain feedback configuration, as drawn above, with a simple lead. Explicitly call out $K_1(s)$, detail how you got to the design (roots locus, bode plots, etc.), robustness (GM, PM, Nyquist), and performance (impulse and step response plots).

b. Assume that you want to implement your controller, $K_1(s)$, digitally, and that you will be using a sample rate of 4Hz (ΔT = 0.25 seconds). Convert your controller design from $K_1(s)$ to $K_1(z)$ using a Tustin mapping($s = \frac{2}{T}\frac{z-1}{z+1}$). Write down the resulting difference equation, and pseudo-code to implement it (assume that you have an interrupt that gets called every ΔT seconds). Be sure to implement any pre-calculation for the next call to reduce the time delay from calculation.

c. Design a new controller, $K_2(s)$, this time explicitly accounting for the ΔT/2 time delay. Do this using a first order Padé approximation. That is, design a compensator for the new plant: $G(s) * \frac{-s+4/\Delta T}{s+4/\Delta T}$. Again, place the (dominant) closed loop poles at the same desired location as in part (a).

d. Again, convert $K_2(s)$ to $K_2(z)$ using the Tustin mapping. Write down the resulting difference equation, and pseudo-code to implement it.

e. Comment on the difference in robustness (Phase and Gain margins) in part (a) vs. part (c). Did you recover the full phase margin by accounting for the delay?

Keep a copy of your solution to question 1 (all parts), you will be continuing to work on this over the next homework.

2. *Discrete Filters*: Consider the following discrete transfer function for a digital filter, implemented at a sample rate of 1Hz:

$$H(z) = \frac{z(z + ½)}{(z - ½)(z + {}^{1}/_{3})}$$

a. Find the difference equation that relates $\boldsymbol{y_k}$ to $\boldsymbol{u_k}$, shift everything up or down until you have a causal relationship (no information from the future).

b. Find the equivalent natural frequency and damping (ω_n and ζ) of this filter. Use the exact mapping of $s = \frac{1}{\Delta T} \ln z$

c. Is the filter stable? Explain.

3. *Distortion based on P-Z mapping*: Given the following lead compensator, K(s), designed to add about 55 degrees of lead in at a frequency of 6 rad/sec (about 1 Hz), use ΔT=0.125 sec:

$$K(s) = \frac{10(s + 2)}{(s + 20)}$$

a. Convert the lead network to its discrete equivalent, $K_{FWD}(z)$, using the "Forward Euler" integration method, and calculate the phase at $z = e^{j\omega T}$, where ω=6. Do it by hand, and then check your results using MATLAB, and show the location of the poles and zeros.

b. Convert the lead network to its discrete equivalent, $K_{BWD}(z)$, using the "Backwards Euler" integration method, and calculate the phase at $z = e^{j\omega T}$, where ω=6. Do it by hand, and then check your results using MATLAB, and show the location of the poles and zeros.

c. Convert the lead network to its discrete equivalent, $K_{TUSTIN}(z)$, using the "Tustin" integration method, and calculate the phase at $z = e^{j\omega T}$, where ω=6. Do it by hand, and then check your results using MATLAB, and show the location of the poles and zeros.

d. Convert the lead network to its discrete equivalent, $K_{MATCHED}(z)$, using the "Matched Pole/Zero" integration method, and calculate the phase at $z = e^{j\omega T}$, where ω=6. Do it by hand, and then check your results using MATLAB, and show the location of the poles and zeros.

e. Plot the magnitude and phase plots (Bode Plots) of parts (a)-(d) along with the continuous system for a frequency of 0.1 to 100 rad/s, and note where they diverge from the continuous system.