University of California, Santa Cruz
Board of Studies in Computer Engineering

CMPE-242:
Applied Feedback Control

Homework #9
Due 20-Mar-2013

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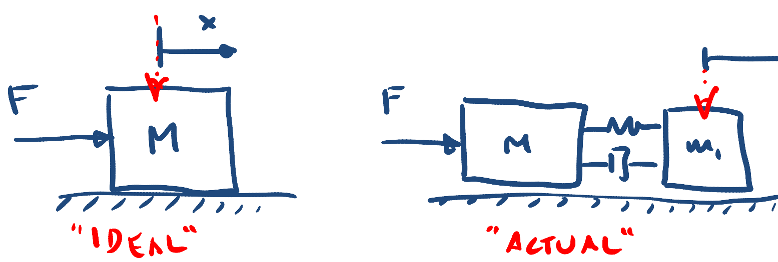
1. *Revisiting Non-collocated Control*: Given the plant below, which describes the input output of a two-mass system, where the displacement of the forward mass is measured. If the spring connecting them was completely rigid, then this would be a $1/s^2$ plant. Since there is, in fact, a spring constant and damping, we wind up with a resonant mode we did not originally know about, which is described in the actual transfer function.

$$G(s) = \frac{10(s + 225)}{s^2(s^2 + 0.225s + 225)}$$



a. Generate a state space realization of this system (by hand, then check in MATLAB).

b. Assume you have full access to the state, ***x***, and design a full state feedback controller to put the dominant poles at $\omega_n = 1$ and $\zeta = 0.5$ (it's the same place you put them in HW#4). Note that you get to choose the locations of two more poles as this is a 4th order system. Put them somewhere "faster" than the dominant ones (but not too fast). What is ***K***? Where are your closed loop poles (use MATLAB)?

c. Use pole placement techniques (acker or place) to design an estimator with very damped poles (use $\zeta = 0.7$) that are in the vicinity of 3-5 times the speed of dominant closed loop poles. What is your gain matrix, ***L***, and where are your closed loop estimator poles?

d. Create the equivalent controller, ***K(s)***, that matched your estimator/controller design. Analyze it both with root locus and bode techniques. Does it look anything at all like what you did in HW#4?

e. Plot the closed loop step response of your system. Make sure that you plot both the output, ***y***, and the control, ***u***. Try going back and changing your non-dominant poles in part (b) and seeing how they affect the step response of ***y*** and ***u***. Is this a big or small change?

f. Redo the controller from part (b), but this time using optimal control (LQR) techniques. Use the cost function: $J = \int_0^{\infty}(\rho y^2 + u^2)dt.$ Tweak p until you get the dominant poles at roughly ω_n = 1 rad/sec. How do the "optimal" pole locations compare with your original HW#4 design, and the design from part (b) above?

g. Redo parts (d) and (e) with the new controller (keep your estimator from before). How do ***y*** and ***u*** compare?

2. *Attitude Stabilization revisited*: You are going to redesign the controller for the non-collocated plant of the satellite model, this time in state space form. We've converted the model for you, and here is the state space version of $\boldsymbol{G_{FORE}(s)}$, which maps the input of the aft thrusters to the fore-body angle:

```
A =[0          0          0          0          0          0
    0    -0.7555    41.9632          0          0          0
    0   -41.9632    -0.7555          0          0          0
    0          0          0    -0.2990    14.9470          0
    0          0          0   -14.9470    -0.2990          0
    0          0          0          0          0    -0.5000]

B = [0.3329    22.9467  110.6833  -85.2094    9.2657    1.9153]^T
C = [87.6846    0.0004   -0.0001   -0.0016   -0.0130  -15.2495]
D = [0]
```

a. Use LQR techniques to pick controller that yields a response similar to what you got on HW#7/8. What is ***K***? Where are your closed loop poles?

b. Add in the state command structure so that you can control to a reference signal. What are your two matrices, $\mathbf{N_u}$ and $\mathbf{N_x}$? Draw the block diagram of the entire control structure.

c. Simulate the closed loop system, plot the step and impulse responses (make sure to include both output and control). Comment on how this compares to your system in HW#7/8.

d. Pick estimator poles that are "faster" than the poles you got in (a) above, but also much slower than your Nyquist frequency, $\omega_s/2$. You will again be using a sample rate of 25Hz. What is your ***L***, where are your closed loop estimator poles? Again, draw the block diagram of the whole structure (including $\mathbf{N_u}$ and $\mathbf{N_x}$).

e. Convert your controller/estimator to a transfer function form, ***K(s)***, and compare it to what you did on HW#8. Does is look the same? Check the compensator on both bode and root locus techniques (extra poles/zeros, etc).

f. Simulate the whole system, for a step and impulse response, and make sure to plot both ***y*** and ***u***.

g. Descretize the controller/estimator to create ***K(z)***, using a sample rate of 25Hz, and simulate it using the simulink files from HW#7/8. ***Note***: if you do this as a transfer function, make sure to carry a whole lot of significant digits, or it won't work.

3. The state space representation we gave you in Problem 2 (above) is a transfer function directly from ***u*** (thrusters) to Θ_{FORE}. In truth, we actually have measurements of both Θ_{AFT} and Θ_{FORE}. This only changes the [C] and [D] matrices, but they are changed to:

```
 A =[0         0         0         0         0         0
     0   -0.7555   41.9632         0         0         0
     0  -41.9632   -0.7555         0         0         0
     0         0         0   -0.2990   14.9470         0
     0         0         0  -14.9470   -0.2990         0
     0         0         0         0         0   -0.5000]

B = [0.3329   22.9467  110.6833  -85.2094    9.2657    1.9153]^T
C = [21.9063   -0.0015    0.0001   -0.0053   -0.0790  -15.4214
     87.6846    0.0004   -0.0001   -0.0016   -0.0130  -15.2495]
D = [0; 0]
```

Repeat problem (2)a-2(f) with the new system, see how things change.