UNIVERSITY OF CALIFORNIA, SANTA CRUZ
BOARD OF STUDIES IN COMPUTER ENGINEERING

CMPE-242:
APPLIED FEEDBACK CONTROL

HOMEWORK #1
DUE 15-JAN-2013

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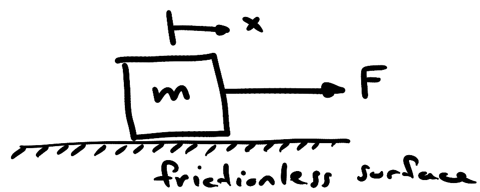
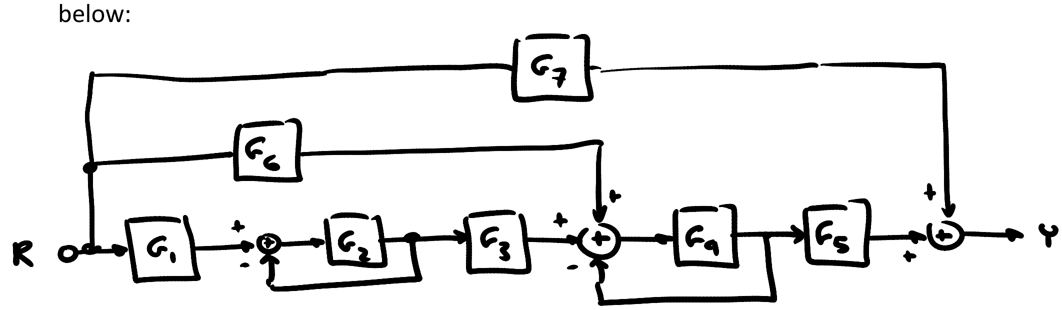
1. *Equations of Motion*: Consider the very simple mechanical system below:



   a. Write down the equations of motion (E.O.M.).
   b. Integrate the equations of motion to find the response to a step input (F(t) = **1**(t)).
   c. Integrate the equations of motion to find the response to an impules (F(t) = δ(t)).
   d. Use the convolution integral to find the response to a step input (meaning: use h(t) from your answer to part (c), and F(t) = **1**(t)).

2. *Block diagram reduction*: Write down the transfer function [Y(s)/U(s)] of the block diagram below:



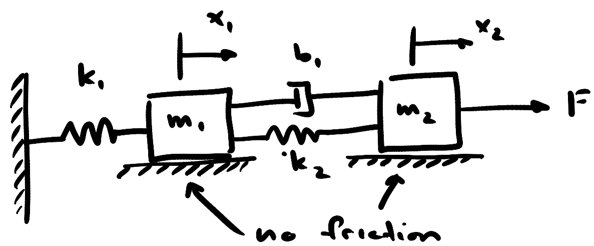
3. *Laplace transform*: Solve the following constant coefficient ordinary differential equations using Laplace Transforms and partial fraction expansion:

   a. $\ddot{y}(t) - 2\dot{y}(t) + 4y(t) = 0$ where $y(0) = 1$ *and* $\dot{y}(0) = 2$

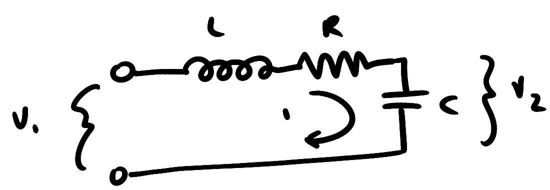
   b. $\ddot{y}(t) + 3y(t) = \sin t$ where $y(0) = 1$ *and* $\dot{y}(0) = 2$

   c. $\ddot{y}(t) + 2\dot{y}(t) = e^{t}$ where $y(0) = 1$ *and* $\dot{y}(0) = 2$

4. *Transfer Functions*: Given the following mass-spring system, derive the transfer function from the position of both of the masses to the forcing function: [$X_2(s)/F(s)$ and $X_1(s)/F(s)$]:



5. *Transfer Functions*: Given the following simple electrical system, derive the transfer function from the input voltage to the output voltage [$V_2(s)/V_1(s)$]:



6. *Dynamic Response*: Given the following third order system:

$$H(s) = \frac{\alpha\omega_n^2}{(s+\alpha)(s^2 + 2\zeta\omega_n s + \omega_n^2)}$$

a. Show that the response to a unit step (U(t)=**1**(t)) is:

$$y(t) = 1 + Ae^{-\alpha t} + Be^{-\sigma t}\,\mathrm{SIN}(\omega_d t - \varphi)$$

where:

$$A = \frac{-\omega_n^2}{(\omega_n^2 - 2\zeta\omega_n\alpha + \alpha^2)}$$

$$B = \frac{\alpha}{\sqrt{(\omega_n^2 - 2\zeta\omega_n\alpha + \alpha^2)(1-\zeta^2)}}$$

$$\varphi = \mathrm{TAN}^{-1}\frac{\sqrt{1-\zeta^2}}{-\zeta} + \mathrm{TAN}^{-1}\frac{\omega_n\sqrt{1-\zeta^2}}{\alpha - \zeta\omega_n}$$

b. Which term dominates as α gets large?
c. Which term dominates as α gets small?
d. Approximate A and B for small values of α.
e. Assume that $\omega_n = 1$ and $\zeta = 0.707$, plot the step response for several values of α. Use MATLAB's *step* command (could you use *impulse*? How?) Comment on where the extra pole becomes unimportant.