

UNIVERSITY OF CALIFORNIA, SANTA CRUZ  
BOARD OF STUDIES IN COMPUTER ENGINEERING



CMPE-242:  
APPLIED FEEDBACK CONTROL

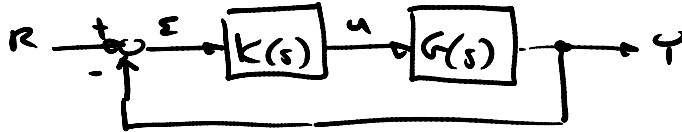
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HOMEWORK #5  
DUE 28-OCT-2010

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1. *Digital and Continuous Equivalents*: Consider the following simple oscillatory system, with two poles on the  $j\omega$  axis:

$$G(s) = \frac{3}{s^2 + 3}$$



- Design a compensator,  $K_1(s)$ , that will place the closed roots of this system at the desired poles of  $s_{des} = -2 \pm 4j$ . Use a unity gain feedback configuration, as drawn above, with a simple lead. Explicitly call out  $K_1(s)$ , detail how you got to the design (roots locus, bode plots, etc.), robustness (GM, PM, Nyquist), and performance (impulse and step response plots).
- Assume that you want to implement your controller,  $K_1(s)$ , digitally, and that you will be using a sample rate of 4Hz ( $\Delta T = 0.25$  seconds). Convert your controller design from  $K_1(s)$  to  $K_1(z)$  using a Tustin mapping ( $s = \frac{2z-1}{Tz+1}$ ). Write down the resulting difference equation, and pseudo-code to implement it (assume that you have an interrupt that gets called every  $\Delta T$  seconds). Be sure to implement any pre-calculation for the next call to reduce the time delay from calculation.
- Design a new controller,  $K_2(s)$ , this time explicitly accounting for the  $\Delta T/2$  time delay. Do this using a first order Padé approximation. That is, design a compensator for the new plant:  $G(s) * \frac{-s+4/\Delta T}{s+4/\Delta T}$ . Again, place the (dominant) closed loop poles at the same desired location as in part (a).
- Again, convert  $K_2(s)$  to  $K_2(z)$  using the Tustin mapping. Write down the resulting difference equation, and pseudo-code to implement it.
- Comment on the difference in robustness (Phase and Gain margins) in part (a) vs. part (c). Did you recover the full phase margin by accounting for the delay?

Keep a copy of your solution to question 1 (all parts), you will be continuing to work on this over the next homework.

2. *Discrete Filters*: Consider the following discrete transfer function for a digital filter, implemented at a sample rate of 1Hz:

$$H(z) = \frac{z(z + 1/2)}{(z - 1/2)(z + 1/3)}$$

- Find the difference equation that relates  $y_k$  to  $u_k$ , shift everything up or down until you have a causal relationship (no information from the future).
  - Find the equivalent natural frequency and damping ( $\omega_n$  and  $\zeta$ ) of this filter.
  - Is the filter stable? Explain.
3. *Distortion based on P-Z mapping*: Given the following lead compensator,  $K(s)$ , designed to add about 55 degrees of lead in at a frequency of 6 rad/sec (about 1 Hz):

$$K(s) = \frac{10(s + 2)}{(s + 20)}$$

- Convert the lead network to its discrete equivalent,  $K_{FWD}(z)$ , using the “Forward Euler” integration method, and calculate the phase at  $Z = e^{j\omega T}$ , where  $\omega=6$ . Do it by hand, and then check your results using MATLAB, and show the location of the poles and zeros.
- Convert the lead network to its discrete equivalent,  $K_{BWD}(z)$ , using the “Backwards Euler” integration method, and calculate the phase at  $Z = e^{j\omega T}$ , where  $\omega=6$ . Do it by hand, and then check your results using MATLAB, and show the location of the poles and zeros.
- Convert the lead network to its discrete equivalent,  $K_{TUSTIN}(z)$ , using the “Tustin” integration method, and calculate the phase at  $Z = e^{j\omega T}$ , where  $\omega=6$ . Do it by hand, and then check your results using MATLAB, and show the location of the poles and zeros.
- Convert the lead network to its discrete equivalent,  $K_{MATCHED}(z)$ , using the “Matched Pole/Zero” integration method, and calculate the phase at  $Z = e^{j\omega T}$ , where  $\omega=6$ . Do it by hand, and then check your results using MATLAB, and show the location of the poles and zeros.
- Plot the magnitude and phase plots (Bode Plots) of parts (a)-(d) along with the continuous system for a frequency of 0.1 to 100 rad/s, and note where they diverge from the continuous system.