

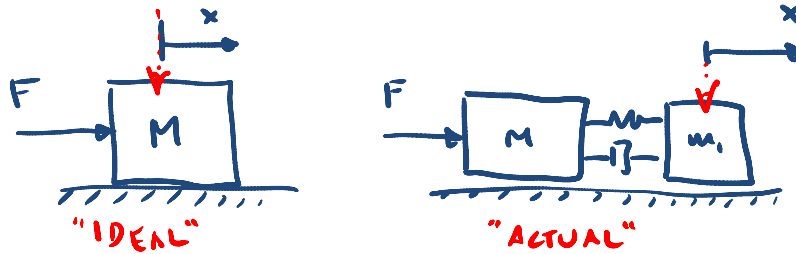


CMPE-242:
APPLIED FEEDBACK CONTROL

HOMEWORK #10
DUE 02-DEC-2010

1. *Revisiting Non-allocated Control*: Given the plant below, which describes the input output of a two-mass system, where the displacement of the forward mass is measured. If the spring connecting them was completely rigid, then this would be a $1/s^2$ plant. Since there is, in fact, a spring constant and damping, we wind up with a resonant mode we did not originally know about, which is described in the actual transfer function.

$$G(s) = \frac{10(s + 225)}{s^2(s^2 + 0.225s + 225)}$$



- Generate a state space realization of this system (by hand, then check in MATLAB).
- Assume you have full access to the state, \mathbf{x} , and design a full state feedback controller to put the dominant poles at $\omega_n = 1$ and $\zeta = 0.5$ (it's the same place you put them in HW#4). Note that you get to choose the locations of two more poles as this is a 4th order system. Put them somewhere "faster" than the dominant ones (but not too fast). What is \mathbf{K} ? Where are your closed loop poles (use MATLAB)?
- Use pole placement techniques (acker or place) to design an estimator with very damped poles (use $\zeta = 0.7$) that are in the vicinity of 3-5 times the speed of dominant closed loop poles. What is your gain matrix, \mathbf{L} , and where are your closed loop estimator poles?
- Create the equivalent controller, $\mathbf{K}(s)$, that matched your estimator/controller design. Analyze it both with root locus and bode techniques. Does it look anything at all like what you did in HW#4?
- Plot the closed loop step response of your system. Make sure that you plot both the output, \mathbf{y} , and the control, \mathbf{u} . Try going back and changing your non-dominant poles in part (b) and seeing how they affect the step response of \mathbf{y} and \mathbf{u} . Is this a big or small change?

- f. Redo the controller from part (b), but this time using optimal control (LQR) techniques. Use the cost function: $J = \int_0^{\infty} (\rho y^2 + u^2) dt$. Tweak ρ until you get the dominant poles at roughly $\omega_n = 1$ rad/sec. How do the “optimal” pole locations compare with your original HW#4 design, and the design from part (b) above?
- g. Redo parts (d) and (e) with the new controller (keep your estimator from before). How do \mathbf{y} and \mathbf{u} compare?
2. *Attitude Stabilization revisited:* You are going to redesign the controller for the non-collocated plant of the satellite model, this time in state space form. We’ve converted the model for you, and here is the state space version of $\mathbf{G}_{FORE}(s)$, which maps the input of the aft thrusters to the fore-body angle:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.7555 & 41.9632 & 0 & 0 & 0 \\ 0 & -41.9632 & -0.7555 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2990 & 14.9470 & 0 \\ 0 & 0 & 0 & -14.9470 & -0.2990 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5000 \end{bmatrix}$$

$$\mathbf{B} = [0.3329 \quad 22.9467 \quad 110.6833 \quad -85.2094 \quad 9.2657 \quad 1.9153]^T$$

$$\mathbf{C} = [87.6846 \quad 0.0004 \quad -0.0001 \quad -0.0016 \quad -0.0130 \quad -15.2495]$$

$$\mathbf{D} = [0]$$

- a. Use LQR techniques to pick controller that yields a response similar to what you got on HW#7/8. What is \mathbf{K} ? Where are your closed loop poles?
- b. Add in the state command structure so that you can control to a reference signal. What are your two matrices, \mathbf{N}_u and \mathbf{N}_x ? Draw the block diagram of the entire control structure.
- c. Simulate the closed loop system, plot the step and impulse responses (make sure to include both output and control). Comment on how this compares to your system in HW#7/8.
- d. Pick estimator poles that are “faster” than the poles you got in (a) above, but also much slower than your Nyquist frequency, $\omega_s/2$. You will again be using a sample rate of 25Hz. What is your \mathbf{L} , where are your closed loop estimator poles? Again, draw the block diagram of the whole structure (including \mathbf{N}_u and \mathbf{N}_x).
- e. Convert your controller/estimator to a transfer function form, $\mathbf{K}(s)$, and compare it to what you did on HW#8. Does it look the same? Check the compensator on both bode and root locus techniques (extra poles/zeros, etc).
- f. Simulate the whole system, for a step and impulse response, and make sure to plot both \mathbf{y} and \mathbf{u} .
- g. Discretize the controller/estimator to create $\mathbf{K}(z)$, using a sample rate of 25Hz, and simulate it using the simulink files from HW#7/8. **Note:** if you do this as a transfer function, make sure to carry a whole lot of significant digits, or it won’t work.

3. Optional Extra Credit: the state space representation we gave you in Problem 2 (above) is a transfer function directly from u (thrusters) to θ_{FORE} . In truth, we actually have measurements of both θ_{AFT} and θ_{FORE} . This only changes the [C] and [D] matrices, but they are changed to:

$$\begin{aligned}
 \mathbf{A} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -0.7555 & 41.9632 & 0 & 0 & 0 \\ 0 & -41.9632 & -0.7555 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0.2990 & 14.9470 & 0 \\ 0 & 0 & 0 & -14.9470 & -0.2990 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.5000 \end{bmatrix} \\
 \mathbf{B} &= [0.3329 \quad 22.9467 \quad 110.6833 \quad -85.2094 \quad 9.2657 \quad 1.9153]^T \\
 \mathbf{C} &= \begin{bmatrix} 21.9063 & -0.0015 & 0.0001 & -0.0053 & -0.0790 & -15.4214 \\ 87.6846 & 0.0004 & -0.0001 & -0.0016 & -0.0130 & -15.2495 \end{bmatrix} \\
 \mathbf{D} &= [0; 0]
 \end{aligned}$$

Repeat problem (2)a-2(f) with the new system, see how things change.