

Homework \#9: Final Review.

1. Re-Read Chapters \#1 through \#6 in FPE (again). Review your notes, and lectures as required.
2. Graduate Students ONLY: Write up a LaTEX (or equivalent) document, detailing the equations of motion for the inverted pendulum, transformation of these equations into a transfer function or state space representation, your design requirements, and your controller designs. Leave the section for experimental results blank. This report is due at the beginning of the final exam.

The following questions are from a practice final. You should be able to do the entire thing in a single three hour set. Try to set some time aside (at least three uninterrupted hours) to attempt it. After doing it, take the time to redo every problem until you have it right and understand it.
3. Consider proportional control (gain stabilization) for the following plant:

$$
G(s)=\frac{3}{(s-1)(s+2)(s+3)}=\frac{3}{s^{3}+4 s^{2}+s-6} .
$$

a. Determine the range of stabilizing $K$ (assuming unity feedback) using Routh-Hurwitz techniques.
b. Plot the root locus of the system (both $180^{\circ}$ and $0^{\circ}$ ), compute break-away points, asymptotes, centroid, $j \omega$-crossings, and stable ranges for $K$.
c. Plot the Bode Plot of the open loop system, denoting any gain and phase margins if they exist.
d. Plot the Nyquist plot of the open loop system, again denoting gain and phase margin (for both negative and positive feedback configurations).

Hint: $\angle \mathrm{G}(j 1)=-180^{\circ}$ and remember that unstable poles have different phase plots than stable ones.
4. Consider the system $K G(s)$ set up in unity feedback: $K G(s)=\frac{K}{(s+2)(s+\alpha)}$. That is:

a. Sketch the root locus with respect to $K$, assuming $\alpha=1$.
b. Sketch the root locus with respect to $\alpha$, assuming $K=10$.

Hint: put the characteristic equation into Evan's Form. Do not bother with the calculation of break-in or break-away points, just provide the basic plot (asymptotes, departure angles, arrival angles, centroid, etc).
5. Find the transfer function, $Y(s) / R(s)$ for the following two systems, and note that there are several ways to compute these transfer functions.

6. Consider a simple inverted pendulum, modeled as: $G(s)=\frac{1}{s^{2}\left(s^{2}-1\right)}$. Assume unity feedback, and design a controller that stabilizes this plant. Hint: Root Locus is probably most helpful (although a Bode design can work as well). Do not spend your time fine tuning the control parameters; just find a compensator strategy that works.
7. Consider the plant: $G(s)=\frac{1}{s\left(s+\frac{4}{3}\right)^{3}}$, in a unity feedback system.
a. Sketch the Root Locus, showing asymptotes, departure angles, etc. Using graphical techniques, find the approximate gain $K$ at the stability boundary (crossing the $\mathrm{j} \omega$-axis).
b. Find a compensator that stabilizes the system with a bandwidth of approximately $10 \mathrm{rad} / \mathrm{s}$ or greater. Find approximate values for any parameters.


