CMPE 240 LINEAR DYNAMIC SYSTEMS

27/JAN/2016

(5) $y \in \mathbb{R}^m$ $\boxed{y = Ax}$ $x \in \mathbb{R}^n$ $m > n$

when everything is working: $y = Ax$

" " " not working: $y \neq Ax$

$B \in \mathbb{R}^{k \times m}$ $By = 0$ for any y which is consistent

$By \neq 0$ for inconsistent.

$y = Ax$ $y - Ax = 0$ consistent $\hat{y} = Ax$

$y - Ax \neq 0$ inconsistent

$By = 0$ for consistent matrix

$BAx = 0$

$\uparrow$

$R(A) \subseteq N(B)$

$By \neq 0 \;\forall\; y \neq Ax$ $\quad$ $By = 0$ implies $By = 0 \;\forall$ consistent

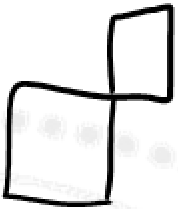
$N(B) \subseteq R(A)$

$$\boxed{N(B) = R(A)}$$

$\text{rank}(B) = m - \text{rank}(A)$

Any B such that $B \cdot A = 0$ and B is full rank

For any matrix $C \rightarrow N(C) \overset{\perp}{+} R(C^T) = n$



$N(B) \perp R(B^T)$

$R(A) \perp N(A^T)$

$+ \quad N(B) = R(A)$

$R(B^T) = N(A^T)$

Basis for $N(A^T)$ "null"

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B = null(A')';
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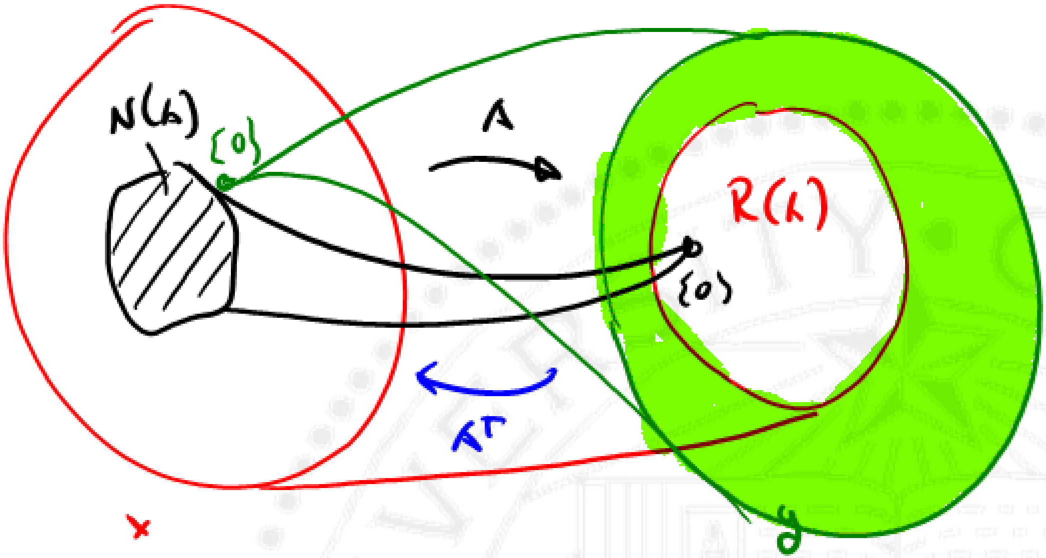
$y = Ax$



$R(A) + N(A^T) = \mathbb{R}^m$

$$A = \left[Q_1 \,\vdots\, Q_2 \right] \begin{bmatrix} R_1 \\ \cdots \\ 0 \end{bmatrix}$$

0

Q_1 orthonormal basis for $R(A)$

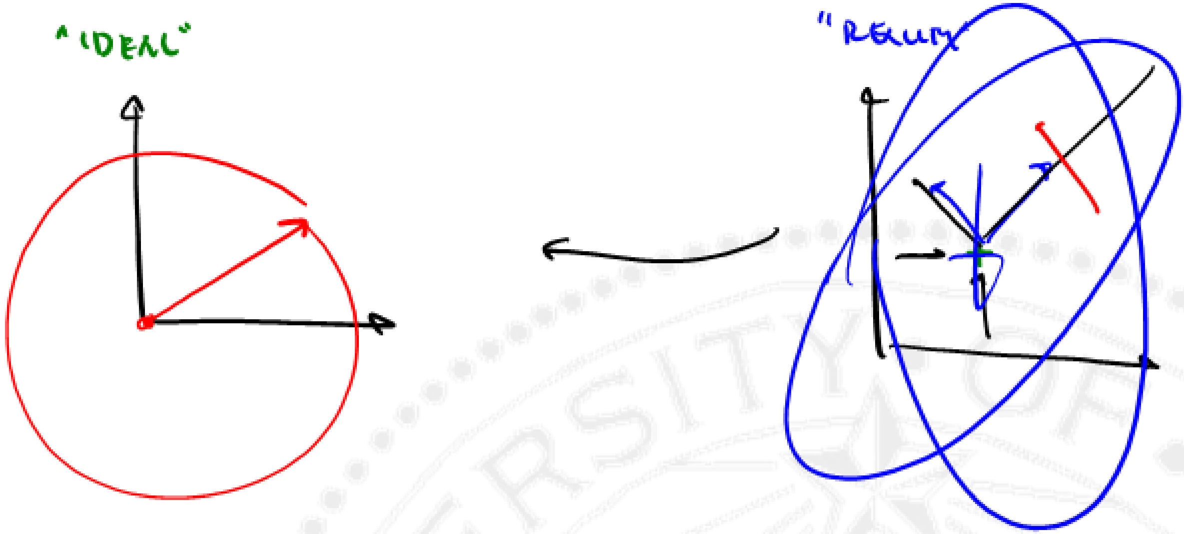
Q_2 orthonormal basis for $N(A^T)$

$[Q, R] = qr(A);$

extract Q_2 either by looking at rank(A) or elements of R

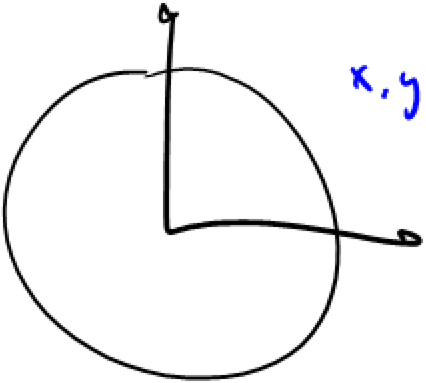
$B = Q_2^T$

$B \cdot A = $



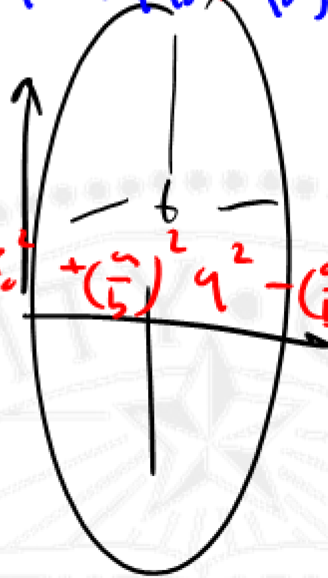
$$h_{meas} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} h_{true} + \underbrace{b_0}_{\text{bias}}$$

$$b^2(x^2 - 2xx_0 + x_0^2) + a^2[y^2 - 2yy_0 + y_0^2] = R^2$$



$x, y.$

$$x^2 - 2xx_0 + x_0^2 + \left(\frac{a}{b}\right)^2 y^2 - \left(\frac{a}{b}\right)^2 2yy_0 + \left(\frac{a}{b}\right)^2 y_0^2 = \frac{R^2}{a^2}$$



$$\left(\frac{x - x_0}{a}\right)^2 + \left(\frac{y - y_0}{b}\right)^2 = R^2$$

$$\frac{2(x-x_0)(y-y_0)}{a^2}$$

$y = A p.$

$\hat{p}$

x_0

y_0

a

b

$$y_{m\times 1} = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} x_{n\times 1} + b_{m\times 1}$$

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$$y(t) = \sum_{\tau=0}^{n-1} u(t-\tau)\, h(\tau)$$

$$z(t) = \sum_{\tau=0}^{m-1} y(t-\tau)\, g(\tau)$$

$$\boxed{\begin{aligned} &\min_{g} \sum_t{}' \left[(g*h)(t)\right]^2 \\ &\text{subj: } (g*h)(D) = 0. \end{aligned}}$$

$$u \longrightarrow \boxed{*h} \xrightarrow{\;y\;} \boxed{*g} \longrightarrow z$$

$$z(t) \approx u(t-D) \qquad z = g*h*u$$

$$(g*h)(t) = \begin{cases} 0 & t \neq D \\ 1 & t = D \end{cases}$$

$$\min \sum_{t \neq D} \left(\sum_{\tau=0}^{m-1} h(t-\tau)\, g(\tau) \right)^2 \quad \text{s.t.} \quad \sum_{\tau=0}^{m-1} h(D-\tau)\, g(\tau) = 1$$

$t \in \{0, \dots, n+m-1\}$

($h \neq 0$ only $\{0, \dots, n-1\}$

($g \neq 0$ only $\{0, \dots, m-1\}$

$(g * h)(t) = c_t^T g \qquad m > n$

- $[h(t) \;\; h(t-1) \;\dots\; h(0) \;\; 0 \;\dots\; 0] \qquad 0 \leq t \leq n$
- $[0 \dots 0 \;\; h(n-1) \;\; h(n-2) \;\dots\; h(0) \;\; 0 \dots 0] \qquad n \leq t < m$
- $[0 \dots 0 \;\; h(n-1) \; h(n-2) \;\dots\; h(t-m+1)] \qquad m \leq t < n+m$

  0 otherwise

$(g*h)(0)$ to $(g*h)(n+m-1)$ $\qquad$ g — $g(0)$ to $g(m-1)$

$$(g*h) = \begin{bmatrix} h(0) & 0 & \cdots & & 0 \\ \vdots & \ddots & & \emptyset & \\ h(n-1) & & \ddots & & 0 \\ 0 & \ddots & & \ddots & \\ & \emptyset & & & h(n-1) \\ & & & 0 & \end{bmatrix} g$$

"toeplitz"

$$\boxed{\min \| Hg - 0 \| \text{ subj to } d^T g = 1}$$

remove D+1 th row from H.

r is soln of $d^T g = 1$

$$r = d(d^T d)^{-1}$$

$Q \longrightarrow N(d^T)$

$$\boxed{g_{ls} = Q(HQ)^{+}(0 - Hr) + r}$$

pseudo-inverse

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(2)

$U \in \mathbb{R}^{n \times k}$

U orthogonal $\langle u_i, u_j \rangle = 0 \quad i \neq j$
$\qquad\qquad\qquad\quad\;\; = 1 \quad i = j$

$$\| U^T x \| \leq \| x \|$$

$$\| U^T x \|^2 = (U^T x)^T (U^T x) = x^T U^{T^T} U^T x = x^T \underbrace{U U^T}_{I} x = x^T x = \| x \|^2$$

U not necessarily square.

$U = \begin{bmatrix} \; \\ \; \\ \; \end{bmatrix}$ skinny $k < n$

$$\left\| \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2 = \| a \|^2 + \| b \|^2$$