Office Hours

CMPE-240

24/FEB/2016

$$f(t) = a e^{-\frac{(t-\mu)^2}{\sigma^2}}$$

ignored

$a, \mu, \sigma \in \mathbb{R}$

amplitude

center

$$p = \begin{bmatrix} a \\ \mu \\ \sigma \end{bmatrix}$$

$t_1 \dots t_N$ $\quad$ $y_1 \dots y_n$

$$E = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(f(t_i) - y_i\right)^2}$$

Gauss-Newton:

$$\Delta p^{(k)} = \begin{bmatrix} \Delta a^{(k)} \\ \Delta \mu^{(k)} \\ \Delta \sigma^{(k)} \end{bmatrix}$$

$$p^{(k+1)} = p^{(k)} + \Delta p^{(k)}$$

$$\min E \longleftrightarrow \min N E^2 = \sum_{i=1}^{N} (f(t_i) - y_i)^2$$

$$f(t) + \left[\nabla_p f(t)\right]^T \Delta p \qquad \nabla_p \triangleq \frac{\partial f}{\partial p}$$

$$f(t) = a e^{-(t-\mu)^2/\sigma^2}$$

$$\frac{\partial f}{\partial a} = e^{-(t-\mu)^2/\sigma^2}$$

$$\frac{\partial f}{\partial \mu} = \frac{2a(t-\mu)}{\sigma^2} e^{-(t-\mu)^2/\sigma^2}$$

$$\frac{\partial f}{\partial \sigma} = \frac{2a(t-\mu)^2}{\sigma^3} e^{-(t-\mu)^2/\sigma^2}$$

$$\min_{\Delta_p} \sum_{i=1}^{N} \Big(\overset{l}{f(t_i)} + \underbrace{\nabla_p f(t_i)}_{A}{}^T \underbrace{\Delta_p}_{\text{"}x\text{"}} - \overset{l}{y_i} \Big)^2$$

$$y = Ax$$

$$\min_{\Delta_p} \sum_{i=1}^{N} \Big[\underbrace{(f(t_i) - y_i)}_{b} + \nabla_p f(t_i)^T \Delta_p \Big]^2$$

$$y = Ax \qquad Ax - y$$

$$\begin{bmatrix} y_1 - f(t_1) \\ y_2 - f(t_2) \\ \vdots \\ y_N - f(t_N) \end{bmatrix} \simeq \overbrace{\begin{bmatrix} \frac{\partial f}{\partial a}\Big|_{t_1} & \frac{\partial f}{\partial \mu}\Big|_{t_1} & \frac{\partial f}{\partial \sigma}\Big|_{t_1} \\ \vdots & & \\ \frac{\partial f}{\partial a}\Big|_{t_N} & \frac{\partial f}{\partial \mu}\Big|_{t_N} & \frac{\partial f}{\partial \sigma}\Big|_{t_N} \end{bmatrix}}^{A} \Delta_p$$

$b = A\,\Delta p^{3\times 1}$

chm — skinny full rank

$\Delta p \simeq \underline{(A^T A)^{-1} A^T b}$

$\Delta p \simeq (A^T A + \mu I)^{-1} A^T b$

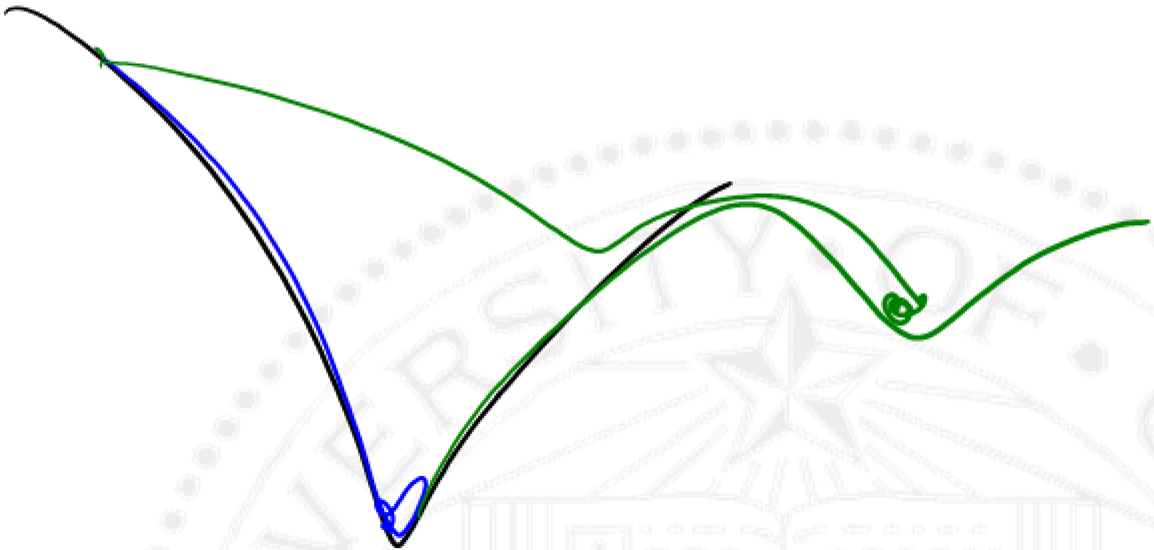
① Initialize $p = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

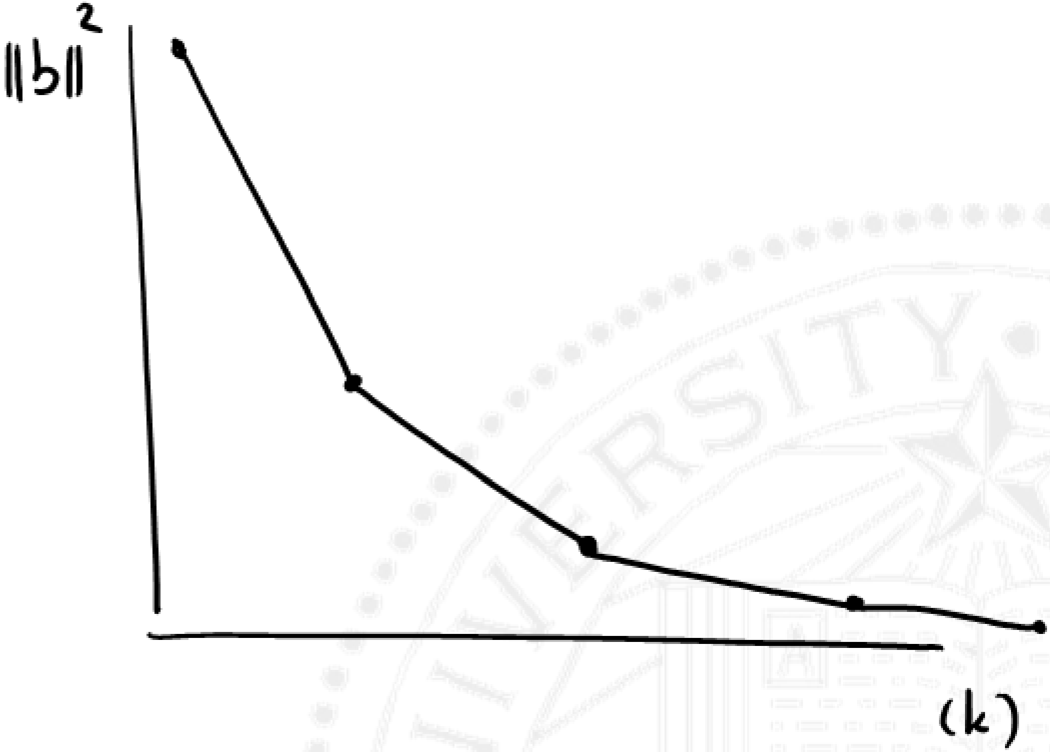
② Form $b = \begin{bmatrix} b_i \\ | \end{bmatrix}$ $\quad b_i = y_i - f(p, t_i)$

③ Form $A = \begin{bmatrix} a_i \end{bmatrix}$ $\quad a_i = \left[\frac{\partial f}{\partial a}\Big|_{t_i} \; \cdots \right]$

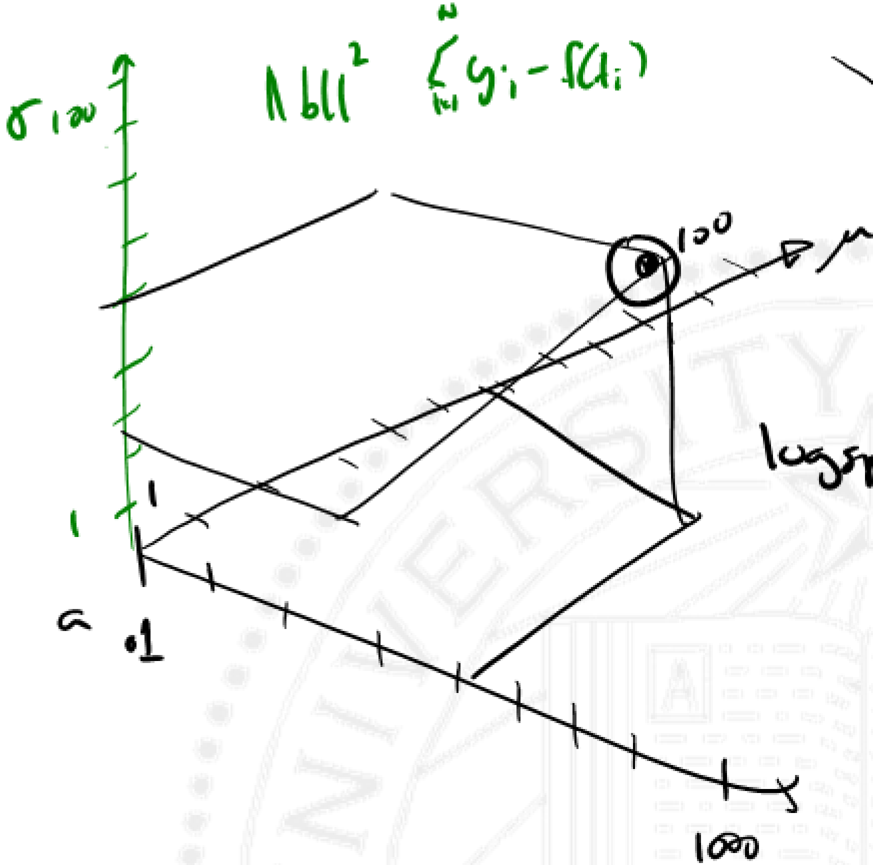
④ Solve LS for Δp $\longrightarrow$ $p^+ = p^- + \Delta p$

⑤ Calculate $\|b\|^2$

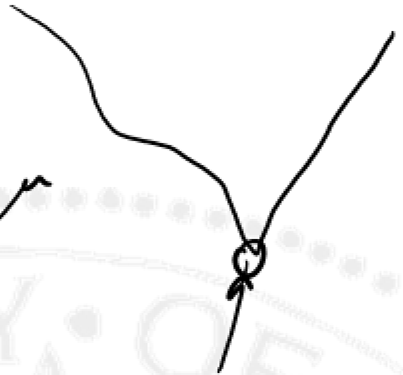


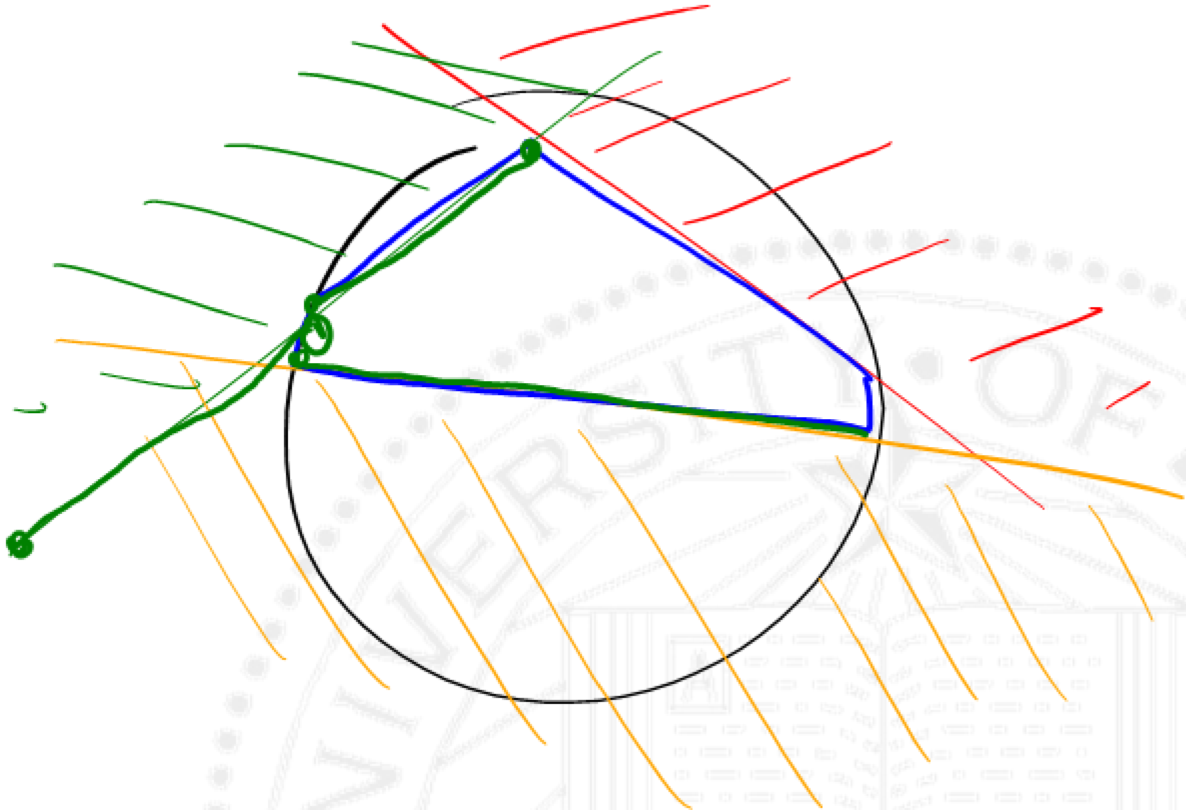


$\|b\|^2$
(k)



$\|b\|^2$ $\sum_{i=1}^{N} (y_i - f(t_i))$
σ 100
1
1
a
.1
100
μ
1000
logspace





eigenvectors Applications

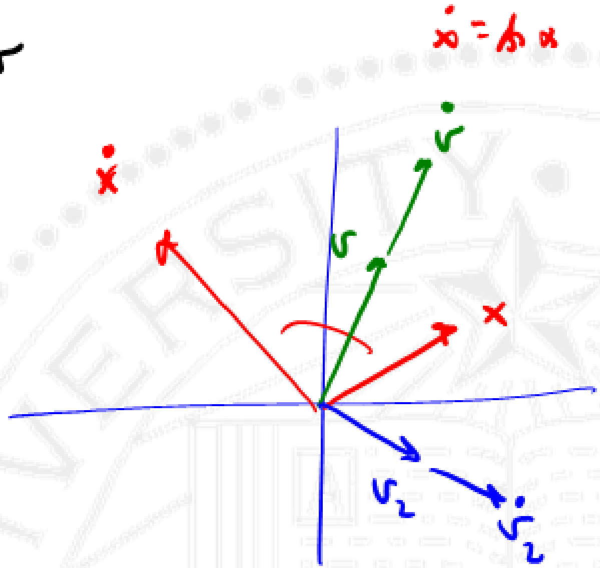
$Av = \lambda v$

$\dot{x} = Ax$ $\quad Av = \lambda v$

$x = v$

$\dot{x} = Av = \lambda v$

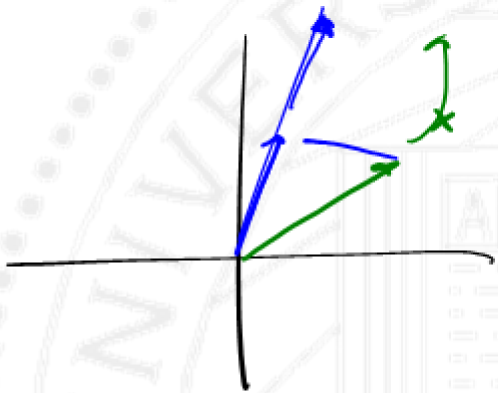
scaling



$w^T A = w^T \lambda \qquad \dot{x} = Ax$

$$w^T \dot{x} = w^T \lambda x$$

$$\underline{w^T \dot{x}} = w^T \lambda x = \lambda \left[\underline{w^T x} \right]$$



$$\underbrace{\begin{bmatrix} v_1 & v_2 & \cdots & v_n \end{bmatrix}}_{V}^{-1} = \underbrace{\begin{bmatrix} w_1^T \\ w_2^T \\ \vdots \\ w_n^T \end{bmatrix}}_{W}$$

$$\dot{x} = \lambda \alpha$$

[illegible]

$$x = a_1 v_1 + a_2 v_2 + \cdots + a_n v_n$$

$$a_i = w_i^T x$$

2

$$\begin{bmatrix} p_1 \cdots p_n \end{bmatrix} \rightarrow \begin{bmatrix} v_1 \cdots v_n \end{bmatrix}$$

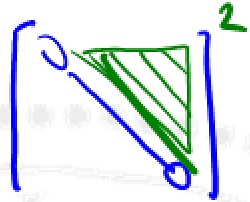
$$\begin{bmatrix} q_1^T \\ \vdots \\ q_n^T \end{bmatrix} \rightarrow \begin{bmatrix} w_1^T \\ \vdots \\ w_n^T \end{bmatrix}$$

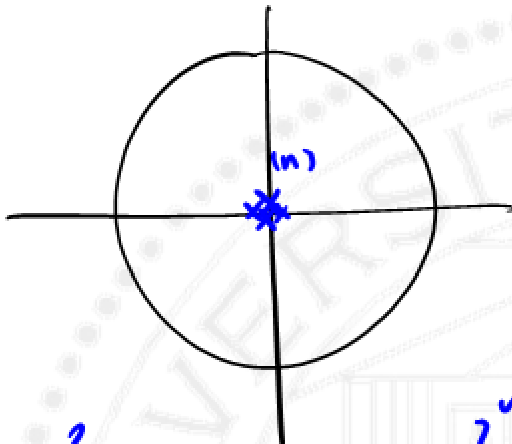
$$(\lambda I - A)^{-1} = \begin{bmatrix} \frac{1}{\lambda-1} & 0 \\ \frac{1}{3}\left(\frac{1}{\lambda-1} - \frac{1}{\lambda+2}\right) & \frac{1}{(\lambda+2)} \end{bmatrix} - \frac{1}{\lambda-1}\begin{bmatrix} & \\ & \end{bmatrix} + \frac{1}{(\lambda+2)}\begin{bmatrix} & \\ & \end{bmatrix}$$

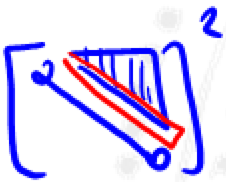
$$\det(zI - A) = z^n \leftarrow \Delta(z).$$

$$\underline{x_{k+1} = A x_k}$$

$$0 = A^n x_0$$





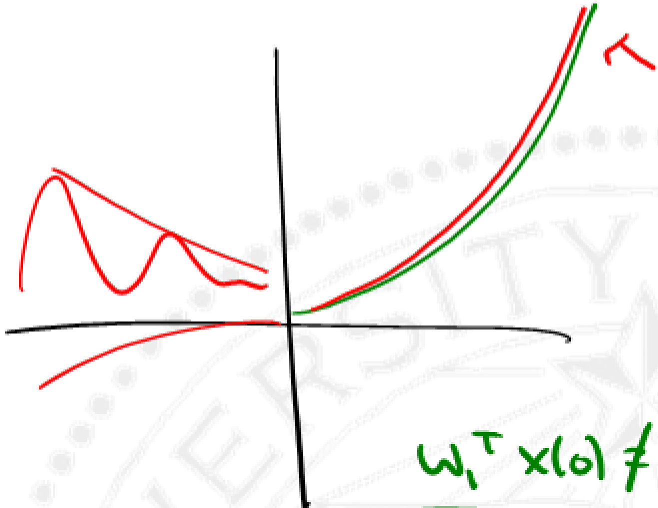


$$z^n = \phi = \det(zI - A)$$

$$X(z) = z^n = 0$$

$$X(A) = A^n = 0$$

$$A^k = A^{k-n} \overset{0}{\cancel{A^n}}$$



$w_1^T x(0) \neq 0.$