Office Hours

19/JAN/2016 CMPE-240.

$f(\cdot): \mathbb{R}^n \to \mathbb{R}^n$

$q = f(p)$

p - price

q - demand

$y_i = \dfrac{\delta q_i}{q_i^*}$ $x_j = \dfrac{\delta p_j}{p_j^*}$

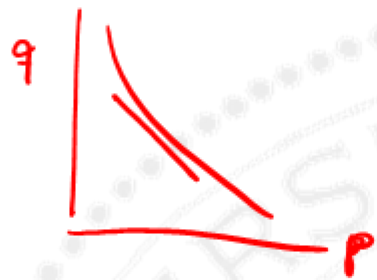
$y = Ex$

$q^* = f(p^*)$

$$q^* + \delta q = f(p^*) + \left.\frac{\partial f}{\partial p}\right|_{p^*} \delta p$$

$$E \to e_{ij} \triangleq \left.\frac{\partial f_i}{\partial p_j}\right|_{p_j^*} \frac{1/q_i^*}{1/p_j^*}$$

e_{ii} → demand for i^{th} good/product vs. price increase of same product given all others staying constant.



$\frac{dq_i}{dp_i} > 0$ "STATUS GOOD"

$$\begin{bmatrix} -1 & e_{12} \\ e_{12} & -1 \end{bmatrix}$$

$e_{12} \triangleq \frac{\text{demand for 1}}{\text{price of 2}}$

SUBSTITUTE → $e_{12} \sim 1$.

COMPLEMENT → $e_{12} < 0$.

$$E = \left[\begin{array}{c|c} -1 & -1 \\ \hline -1 & -1 \end{array}\right]$$ perfectly complementary.

$N(E)$ $\quad x = \alpha\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$y = Ax$

$$\begin{array}{cc} & \leftarrow m \rightarrow \\ n & \begin{bmatrix} \\ A \\ \\ \end{bmatrix} \end{array}$$

$\hat{y} = Ax + v$

noise

pinv(A)

xhat = A\y.

normal equations

Find $\hat{x}$, given y.

$$A^T \hat{y} = \underbrace{A^T A}_{\mathbb{R}^m} \overset{\mathbb{R}^n}{x} + A^T v$$

$\mathbb{R}^m$ $\mathbb{R}^m$

$\hat{x} \approx x$.

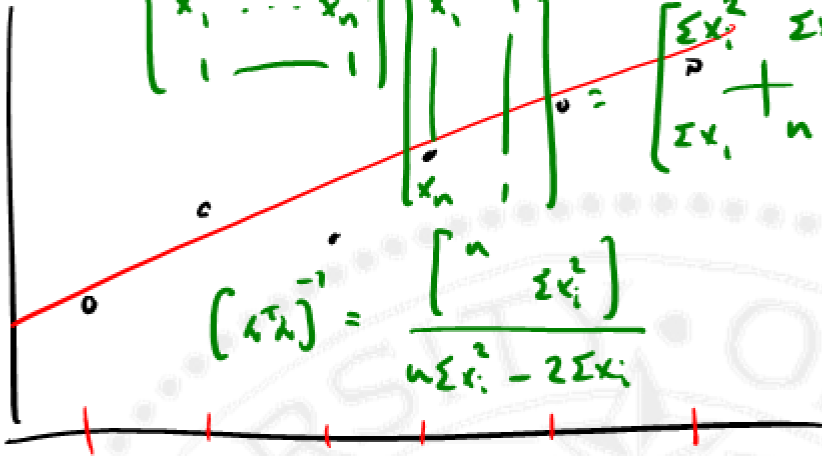
$$\boxed{\tilde{x} = [A^T A]^{-1} A^T y.}$$

LEAST SQUARES SOLUTION TO $y = Ax$.

$$\underbrace{[A^T A]^{-1} [A^T A]}_{I} x = [A^T A]^{-1} \left[A^T \hat{y} - A^T v \right]$$

$$x = [A^T A]^{-1} A^T y - \underbrace{[A^T A]^{-1} A^T v}_{\text{noise}}$$

$$\begin{matrix} x_1 & y_1 \\ \vdots & \vdots \\ x_n & y_n \end{matrix}$$

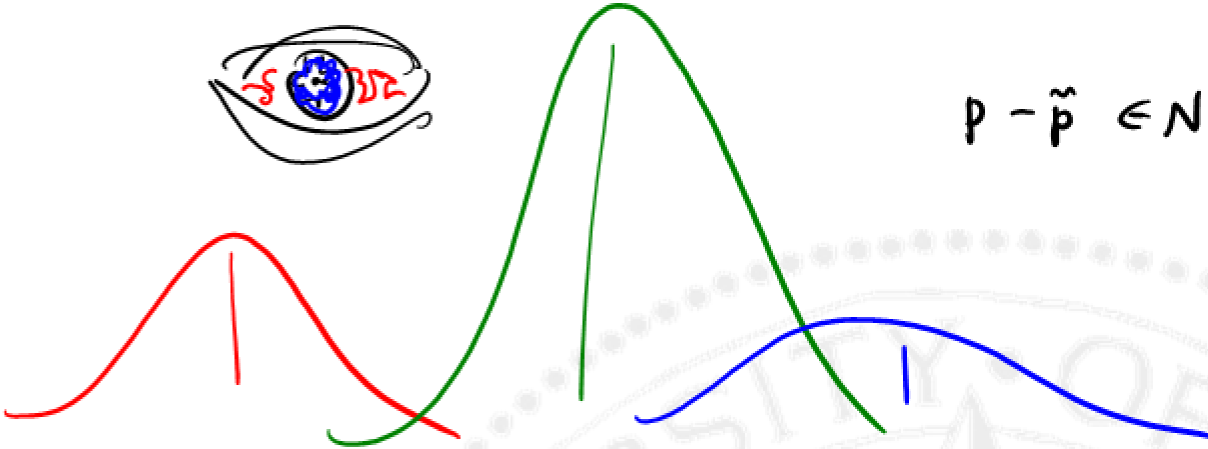


$$\begin{bmatrix} x_1 & \cdots & x_n \\ 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix} = \begin{bmatrix} \Sigma x_i^2 & \Sigma x_i \\ \Sigma x_i & n \end{bmatrix}$$

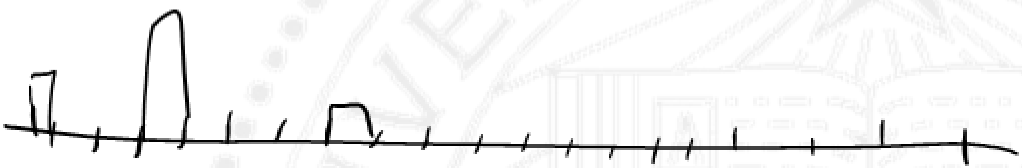
$$(A^T A)^{-1} = \frac{\begin{bmatrix} n & \\ & \Sigma x_i^2 \end{bmatrix}}{n\Sigma x_i^2 - 2\Sigma x_i}$$

$$y = mx + b \longrightarrow y_i = mx_i + b \qquad \begin{bmatrix} \hat{m} \\ b \end{bmatrix} = (A^T A)^{-1} A^T \begin{bmatrix} y_1 \\ y_n \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \underbrace{\begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix}}_{\text{skinny } A} \begin{bmatrix} m \\ b \end{bmatrix}$$



$p - \tilde{p} \in N(k)$



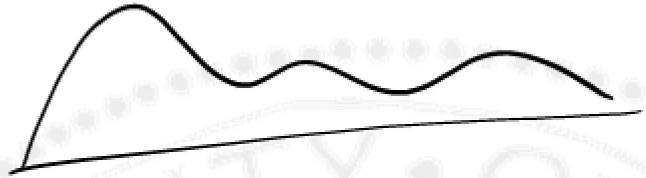
$p \in \mathbb{R}^{20 \times 1}$

$\downarrow \quad\quad \downarrow$

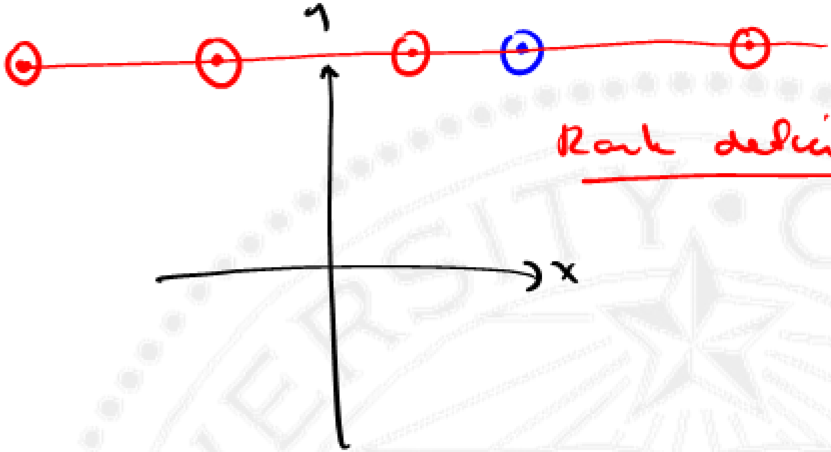
$r \quad\quad \tilde{r}$

$p \quad\quad \tilde{p}$

$\cancel{x} = I \mapsto \rho$



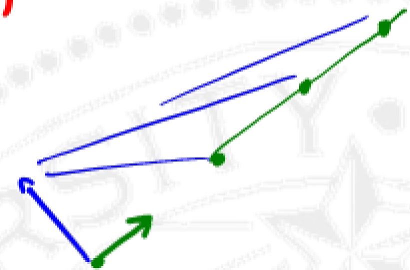
$$\begin{bmatrix} A \end{bmatrix} \underbrace{R I}_{\begin{bmatrix} r_{11} & & 0 \\ & \ddots & \\ 0 & & r_{nn} \end{bmatrix}}$$



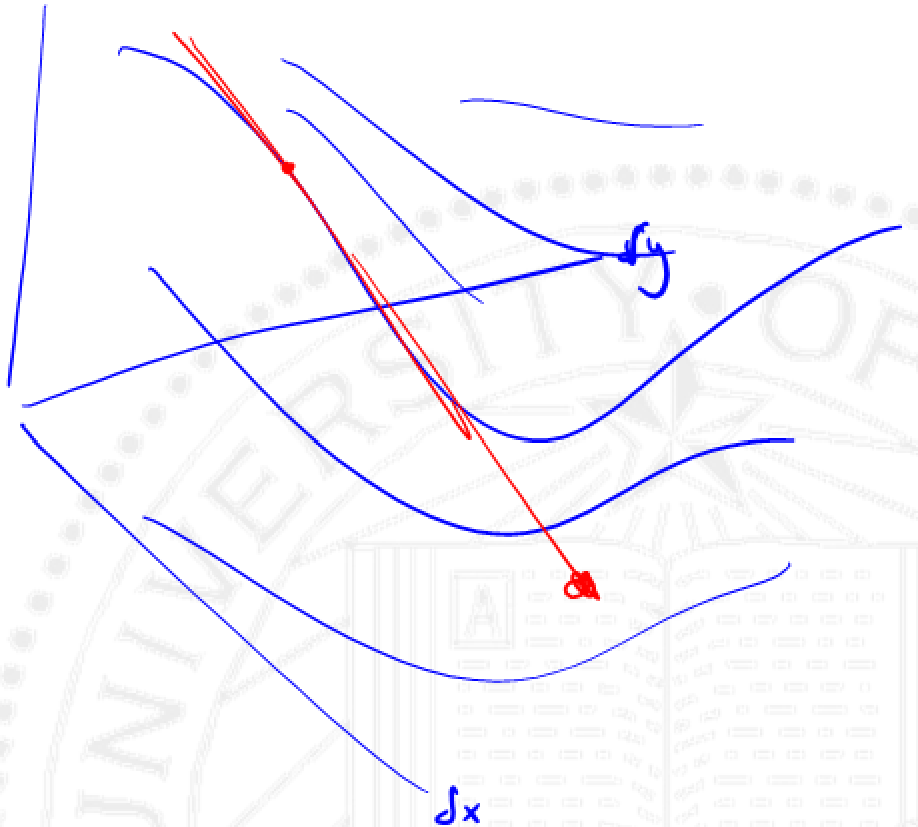
Rank deficient

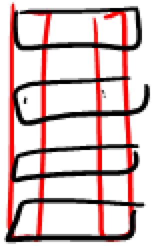
measurement

$$\begin{bmatrix} p_1 \\ \vdots \\ p_4 \end{bmatrix} = \underbrace{\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}}_{A} \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix}$$

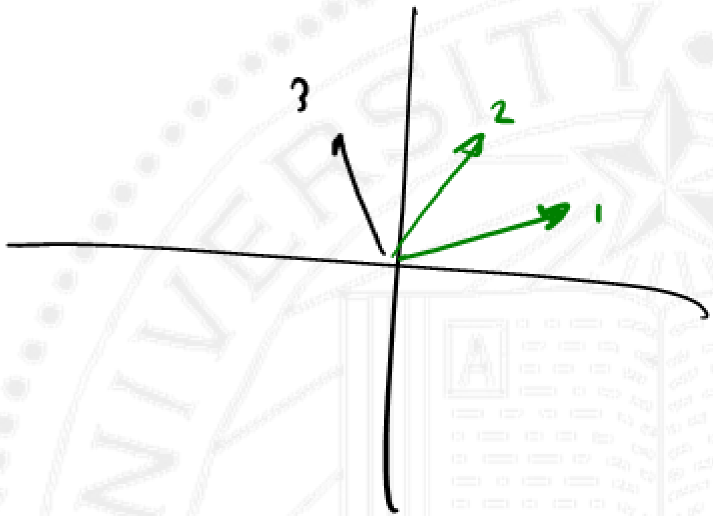


$$\mathbb{R}^2 \rightarrow \mathbb{R}^4$$


δy
δx



Rank(2)



$[0, 1]$ $\qquad x = [1 \; 0 \; 1]$

$$y = [1\;1\;1\;\;0\;0\;0\;\;1\;1\;1]$$

$$y = Ax = \underbrace{\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}}_{A} x$$

we minimize $B \Rightarrow BA = I$

$$\hat{x} = By = BAx = x$$

$$\hat{x} = By = B(Ax+v) = \underbrace{BAx}_{I} + \underbrace{Bv}_{}$$

$\mathbb{R}^m$

if $v \in N(B) \longrightarrow \{v \mid v \in N(B)\} \overset{?}{\longrightarrow}$

$\{\tilde{v} \mid v \notin N(B)\} \longrightarrow$ | noise that gets through.

$v \in N(B)$ $\quad \boxed{\alpha v} \in N(B)$

all noise to which you are immune.