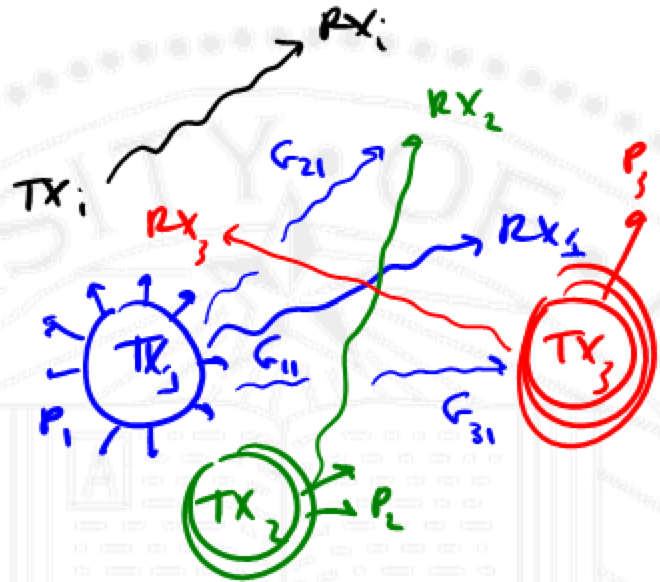
CMPE-240 Linear Dynamical System (LDS)

11-JAN 2016

n - TX↔RX pairs

$TX_i \longrightarrow (P_i) \longrightarrow$



$$s_i = G_{ii} P_i$$

↑

TX → RX GOOD

$$q_i = \sigma + \sum_{i \neq j} G_{ij} P_j$$

$$SNR_i \triangleq s_i / q_i > \gamma$$

$SNR_i(t) \doteq \dfrac{s_i(t)}{q_i(t)} = \alpha\gamma \qquad \alpha > 1.$

$\overset{\shortparallel}{S_i(t)}$

$p_i(t+1) = p_i(t)\left(\alpha\gamma / \underset{\overset{\shortparallel}{S_i(t)}}{SNR_i(t)}\right)$

(a) Show that this is an LDS.

$p(t+1) = Ap(t) + b \quad \leftarrow$ affine problem

$A \in \mathbb{R}^{n \times n} \quad b \in \mathbb{R}^n$

$$p_i(t+1) = \frac{\alpha\gamma p_i(t)}{S_i(t)} = \frac{\alpha\gamma p_i(t)}{S_i(t)/q_i(t)} = \frac{\alpha\gamma p_i(t) q_i(t)}{S_i(t)}$$

$$= \frac{\alpha\gamma \cancel{p_i(t)} \left[\sigma + \sum_{i \neq j} G_{ij} p_j(t)\right]}{G_{ii} \cancel{p_i(t)}}$$

$$= \boxed{\frac{\alpha\gamma \left[\sigma + \sum_{i \neq j} G_{ij} p_j(t)\right]}{G_{ii}}}$$

$$
\underbrace{\begin{bmatrix} p_1(t+1) \\ p_2(t+1) \\ \vdots \\ p_n(t+1) \end{bmatrix}}_{p(t+1)} = \underbrace{\begin{bmatrix} 0 & \frac{\alpha\gamma G_{12}}{G_{11}} & \frac{\alpha\gamma G_{13}}{G_{11}} & \cdots & \frac{\alpha\gamma G_{1n}}{G_{11}} \\ \frac{\alpha\gamma G_{21}}{G_{22}} & 0 & \frac{\alpha\gamma G_{23}}{G_{22}} & \cdots & \frac{\alpha\gamma G_{2n}}{G_{22}} \\ \vdots & & & & \vdots \\ \frac{\alpha\gamma G_{n1}}{G_{nn}} & \frac{\alpha\gamma G_{n2}}{G_{nn}} & \cdots & \frac{\alpha\gamma G_{n(n-1)}}{G_{nn}} & 0 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} p_1(t) \\ p_2(t) \\ \vdots \\ p_n(t) \end{bmatrix}}_{p(t)} + \underbrace{\begin{bmatrix} \frac{\alpha\gamma\sigma}{G_{11}} \\ \frac{\alpha\gamma\sigma}{G_{22}} \\ \vdots \\ \frac{\alpha\gamma\sigma}{G_{nn}} \end{bmatrix}}_{b}
$$

$$
\alpha\gamma\left[\tilde{A} + b\right]
$$

$\underline{p}(0) \quad A \quad b$

$\underline{p}(1) = A p(0) + b$

$\underline{p}(2) = A p(1) + b$

$\vdots$

$\rightarrow p = [p \; p_{new}]$

diag(. . .) $\quad \begin{bmatrix} \ddots \end{bmatrix}$

③ $f: \mathbb{R}^n \to \mathbb{R}^m$ its linear

$$\left.\begin{array}{l} \alpha f(x) = f(\alpha x) \\ f(x+y) = f(x) + f(y) \end{array}\right\} \text{linear}$$

Show that there is a matrix $A \in \mathbb{R}^{m \times n} \to f(x) = Ax$.

$A_{ij} \leftarrow ?$ is A unique $f(x) = \tilde{A}x \to A = \tilde{A}$

$x \triangleq \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n$ $\qquad x = \underbrace{x_1}_{\text{scalar}} e_1 + x_2 e_2 + \dots + x_n e_n$ $\qquad e_j \triangleq \begin{array}{c} \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \\ \leftarrow j\text{th} \\ \\ \\ \end{array}$

$e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ $\quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ $\quad e_n = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$

//

$f(\cdot)$ is linear

$$f(x) = x_1 f(e_1) + x_2 f(e_2) + \dots + x_n \underbrace{f(e_n)}_{\in \mathbb{R}^m}.$$

$$\underbrace{f(x)}_{\mathbb{R}^m} = \underbrace{\begin{bmatrix} f(e_1) & f(e_2) & \dots & f(e_n) \end{bmatrix}}_{A \in \mathbb{R}^{m \times n}} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

$f(\underline{x}) = A\,\underline{x}.$

suppose $\tilde{A} \in \mathbb{R}^{m \times n} \rightarrow f(x) = \tilde{A}x$ $\quad \tilde{A}x = Ax$

$\underline{(A - \tilde{A})x = \phi \quad \forall\, x \in \mathbb{R}^n}$

$(A - \tilde{A})e_1 = \phi$ —

$\vdots$

e_n

$\underline{A = \tilde{A}.}$