Office Hours

02/MAR/2016

CMPE 240 — Position Definite / Semi-definite

$$\dot{x} = \overset{n \times n}{A} x + \overset{n \times 3}{B} \overset{3 \times 1}{u}$$

$$\underset{3 \times 1}{y} = \underset{3 \times n}{C} x + \underset{3 \times 3}{D} \underset{3 \times 1}{u}$$

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$C\dot{v} + Gv = Fu \qquad y = Kv$

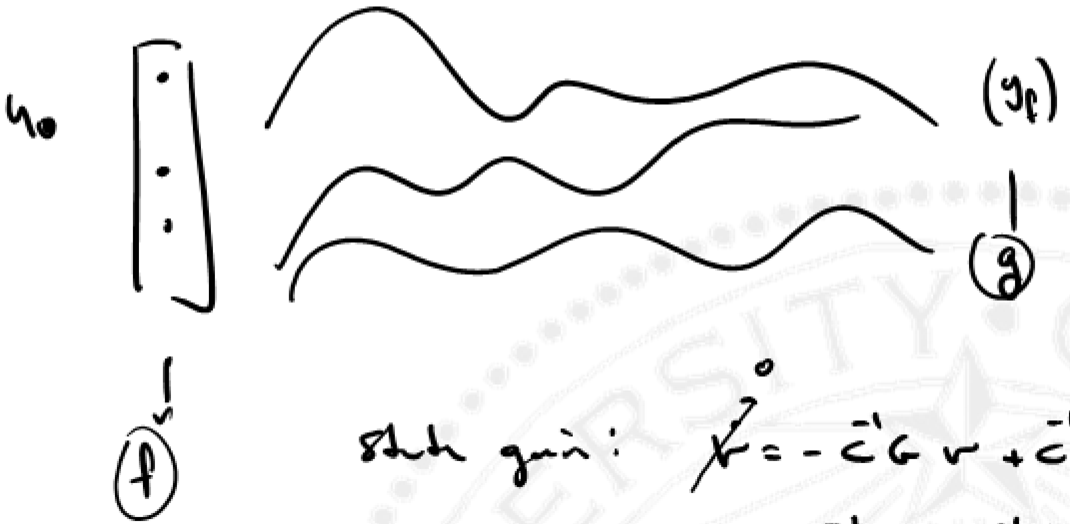
$\dot{x} = Ax + Bu \qquad C^{-1}C\dot{v} = -Gv + Fu$

$y = Cx + Du$

$\dot{v} = \underbrace{-C^{-1}G}_{A}v + \underbrace{C^{-1}F}_{B}u \qquad u_i \in \{0, 1\}$

$y = \underbrace{K}v + \underset{3\times 3}{\underset{T}{\phi}}\,\underline{u}$

<u>$t < 0$</u> <u>shti</u> $u(1) = f$



Stdt gain: $\cancel{\dot{x}}^{0} = -C^{-1}Gv + C^{-1}F\cancel{x}^{f}$

$$0 = -C^{-1}Gv + C^{-1}Ff$$

$$\cancel{C}C^{-1}Gv = \cancel{C}C^{-1}Ff$$

$$Gv = Ff \therefore v = \underline{G^{-1}F}f.$$

$$\boxed{y_0 = K\underline{G^{-1}F}f} \qquad \boxed{y_F = KG^{-1}Fg}$$

$$G^{-1}F = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \qquad KG^{-1}F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{aligned} \dot{v} &= -C^{-1}G v + C^{-1}F u \\ y &= K v \end{aligned}\right]$$

$y_0 = f \leftarrow$

$y_f = g \leftarrow$

$$f = \begin{bmatrix} \{0 & 1\} \\ \{0 & 1\} \\ \{0 & 1\} \end{bmatrix} \qquad g = \begin{bmatrix} \{0 & 1\} \\ \{0 & 1\} \\ \{0 & 1\} \end{bmatrix}$$

Brute Force:

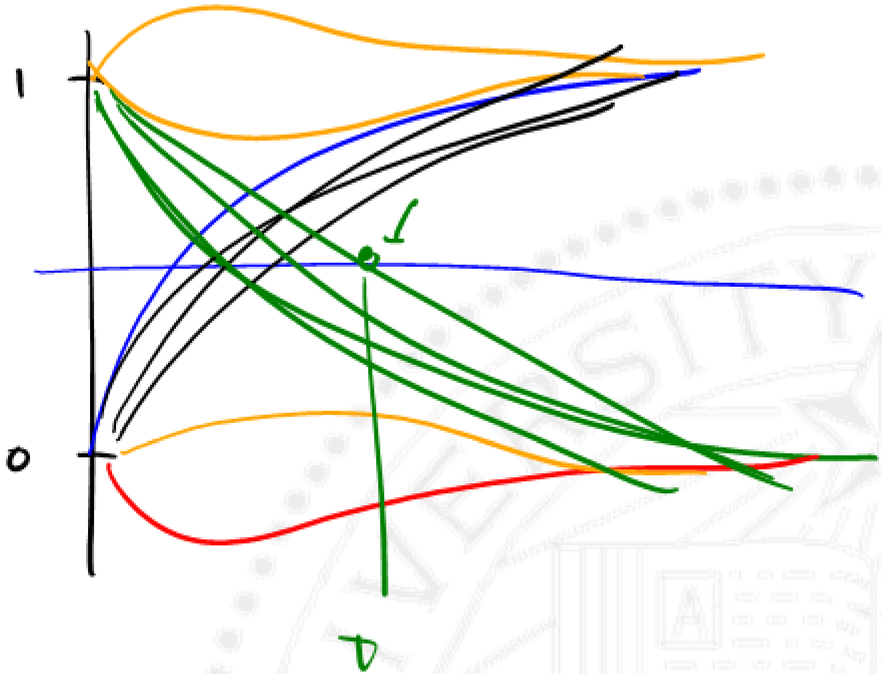
time vector $[0:0.01:10]'$;

$\text{lsim}(sys, u, T, X_0)$

$$u = \begin{bmatrix} g_1 + T*0 & g_2 + T*0 & g_3 + T*0 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}$$

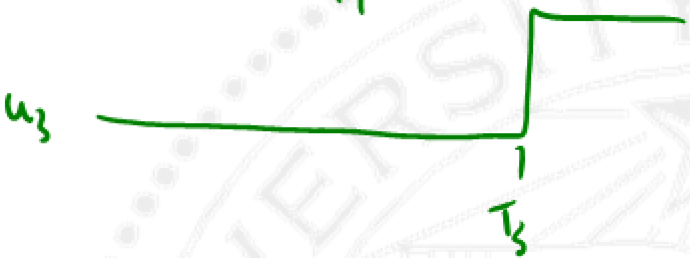
$u = [g_1 * \text{ones}(\text{length}(T),1) \;\; g_2 * \text{ones}(\text{length}(T),1) \;\; g_3 \cdots$

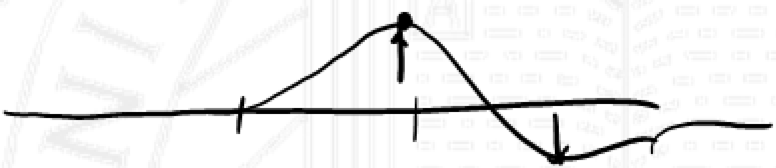
$sys = ss(A, B, C, D)$


1
0
τ

(b)

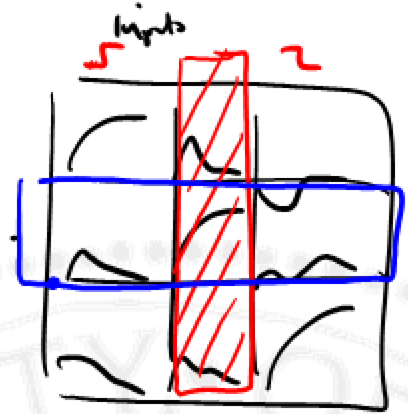
$u_2 = 0 \;\forall\; t$



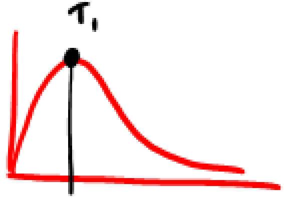


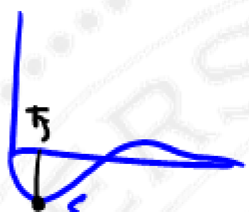
$$y_2(t) = \pm s_{21}(t-\tau_1) \pm s_{23}(t-\tau_3)$$

outputs



$s(t)$.

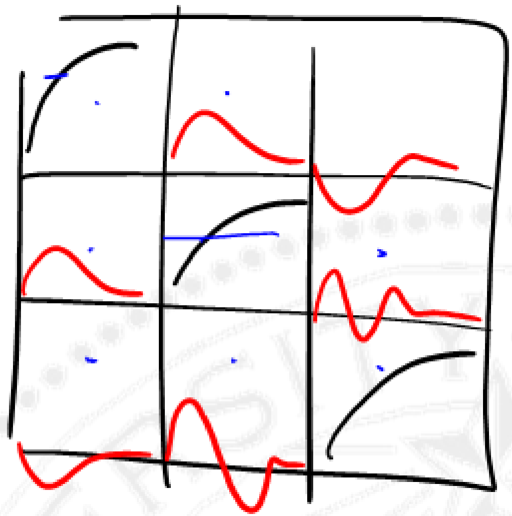




s_{ij}

inputs

outputs



Σ errors rms

> 0.5

sys = ss(...)

[Y, T] = step (sys)

10

SVD → $A = U\Sigma V^T$ $\quad U^TU = I \quad V^TV = I$

not necessarily square

$$\Sigma = \begin{bmatrix} \sigma_1 & \cdots & 0 \\ 0 & \ddots & \sigma_n \end{bmatrix}$$

singular values

$U \stackrel{\triangle}{=}$ left singular vectors

$[u_1\ u_2 \ldots u_n]$

V right singular vector

$\begin{bmatrix} v_1\ v_2 \ldots v_m \end{bmatrix}$

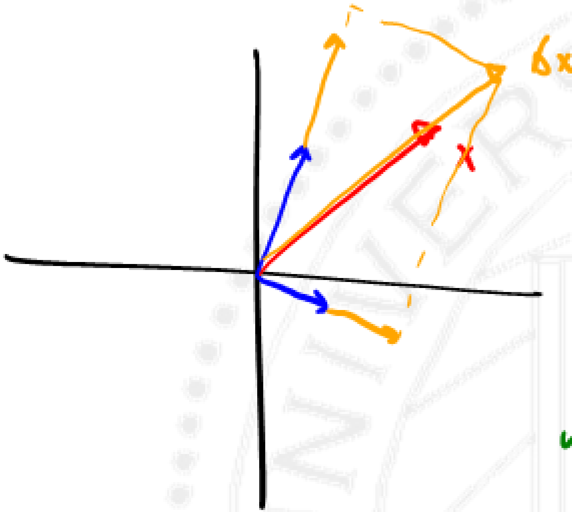
$\sigma_1 > \sigma_2$

$A = U \Sigma V^T$

$y = Ax$

$$x \rightarrow \boxed{V^T} \rightarrow \boxed{\Sigma} \rightarrow \boxed{U}$$

$y = \tilde{w}_i^T \lambda_i v_i$



$z(0) = x \in \mathbb{R}^n$

$w(t) = A\, z(t)$

$z(t+1) = A^T w(t)$

$\dfrac{w(t)}{\|w(t)\|} \approx u_1$

$\dfrac{z(t)}{\|z(t)\|} \approx v_1$

$w(t) = A z(t) \longrightarrow w_k = A z_k \qquad z_{k+1} = A^T w_k$

$w_{k+1} = A z_{k+1} = A A^T w_k \qquad w_k = (AA^T)^k w_0$

$z_{k+1} = A^T w_k = A^T A z_k \qquad z_k = (A^T A)^k z_0$

$SVD(A) = U \Sigma V^T$

$\Sigma = \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_n \end{bmatrix}$

$w_k = (U \Sigma \underbrace{V^T \cdot V}_{I} \Sigma U^T)^k w_0$

$w_k = (U \Sigma^2 U^T)^k w_0$

$$\boxed{w_k = U \Sigma^{2k} U^T w_0}$$

$$w_k = \sum_{i=1}^{n} \sigma_i^{2k} (u_i^T w_0) u_i$$

$$\uparrow$$

$$\sigma_1$$

$$\frac{w_k}{\|w_k\|} = \frac{\sigma_1^{2k} (u_1^T w_0) u_1}{\| \downarrow \|} = \pm u_1$$

$$\approx \underline{\sigma_1^{2k} (v_1^T w_0) u_1}$$

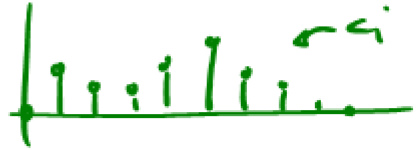
Fails if $v_1^T w_0 = \phi$.

$$U^T U = U U^T = I$$

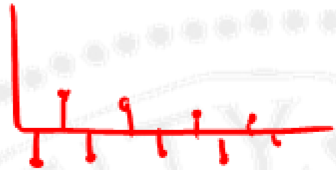
$$V^T V = V V^T = I$$

(b)

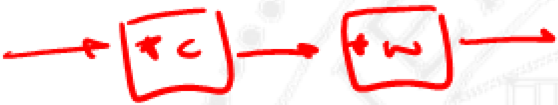
$c_1, c_2 \cdots c_n$



$w_1, w_2 \cdots w_n$



← STEMPLOT



$$E_{TOT} = \sum_{i=2}^{2n} h_i^2$$

$$h_i = \sum_{j=1}^{i-1} w_j c_{i-j} \qquad i = 2, \ldots, 2n$$

$$E_{dn} = \sum_{i=n+1-k}^{n+1+k} h_i^2$$

$$h = Aw$$

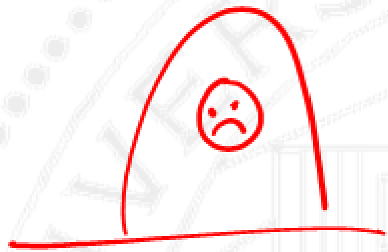
$$\begin{bmatrix} h_1 \\ \vdots \\ \vdots \\ h_{2n} \end{bmatrix} = \begin{bmatrix} c_1 & 0 & \cdots & 0 \\ c_2 & c_1 & 0 \cdots & 0 \\ \vdots & & & \\ c_n & c_{n-1} & \cdots & c_1 \\ & c_n & & \\ 0 & & \ddots & \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ \vdots \\ w_n \end{bmatrix}$$

$$E_{tot} = \|h\|^2 = h^T h = w^T A^T A w$$

$$\bar{h} = \begin{bmatrix} h_{n+1-k} \\ \vdots \\ h_{n+1+k} \end{bmatrix}$$

$$\bar{h} = Bw$$

ratio: $\boxed{\dfrac{w^T B^T B w}{w^T A^T A w}}$ if $A^T A = I$.



① $\dot{x} = Ax + Bu$ $\qquad x_0 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

$y = Cx + Du$

(1) $u(t) = e^{-t}$ $\qquad y(t) = e^{-3t} + e^{-2t}$ $\rightarrow Y(s) = \frac{1}{s+3} + \frac{1}{s+2}$

(2) $u(t) = e^{-3t}$ $\qquad y(t) = 3e^{-3t} - e^{-2t}$ $\qquad Y(s) = \frac{3}{s+3} + \frac{-1}{s+2}$

$$Y(s) = H(s)U(s) + Y_{ic}(s)$$

$H(s)$: $C[sI-A]^{-1}B + D$

$Y_{ic}(s)$: $C[sI-A]^{-1}x_0$

$$Y_1(s) = H(s)\,U_1(s) + Y_{ic}(s) =$$

$$Y_2(s) = H(s)\,U_2(s) + Y_{ic}(s) =$$

$$Y_1(s) - Y_2(s) = H(s)\left[U_1(s) - U_2(s)\right] + \phi .$$

$$C[sI-A]^{-1}B + D \leftarrow \text{unique}$$

not unique

$$T^{-1}AT \mid T^{-1}B \mid CT \mid D$$

Find T such that $$T^{-1}AT = \Lambda = \begin{bmatrix} \lambda_1 & & \\ & \lambda_2 & \\ & & \lambda_3 \end{bmatrix}$$

3x3

?

ss2tf   tf2ss