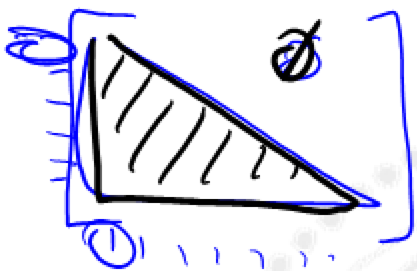
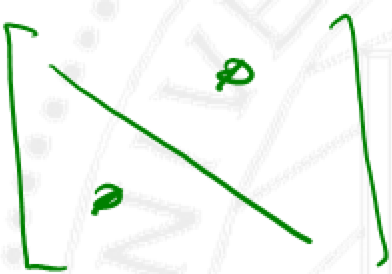
Interpretations of a_{ij}

y_i depends on $x_1 \cdots x_i$



$y_i = f(x_j)$ $i=j$



a_{14}

Linear Elastic Structure

Steel Frame Building

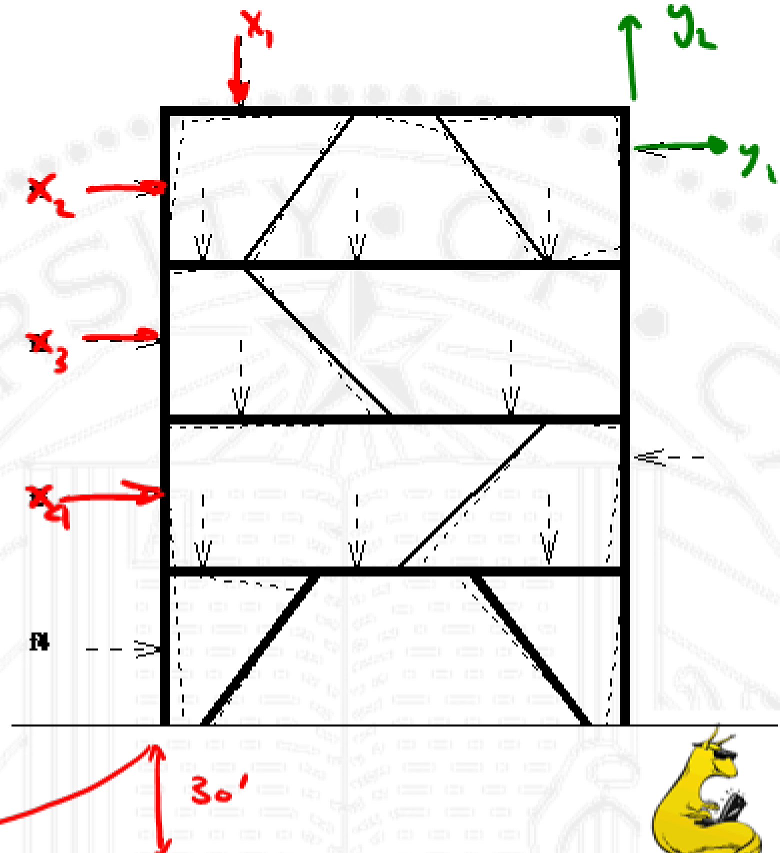
x_j: external force applied to some node or in a fixed direction

y_i: displacement of a given node in some fixed direction

$$y = Ax$$

a_{ij} — m/N

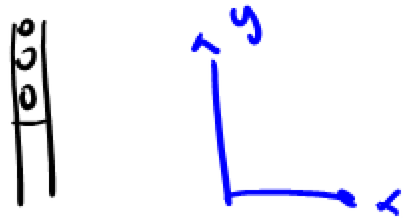
A - "compliance matrix"

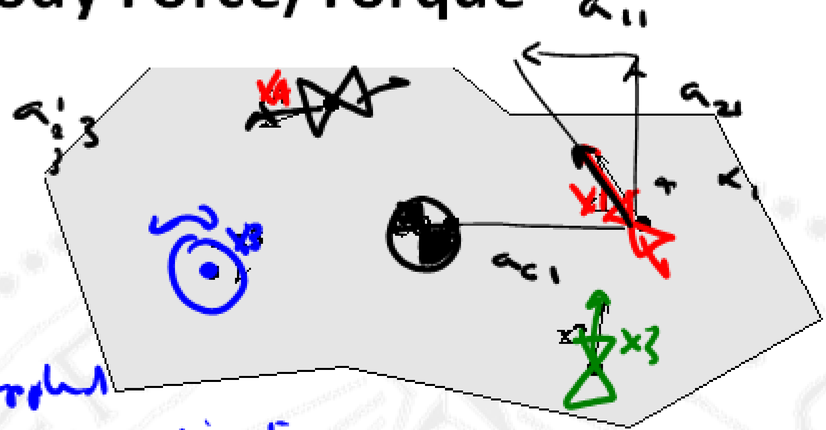


$[y] = x_6 [\quad] [x]$

$y = Ax$

Rigid Body Force/Torque





x_j external force, torque applied at some point and in some direction

$y_i \in \mathbb{R}^6$ resulting total force & torque on the center of mass.

$y_1 \; y_2 \; y_3$ — forces

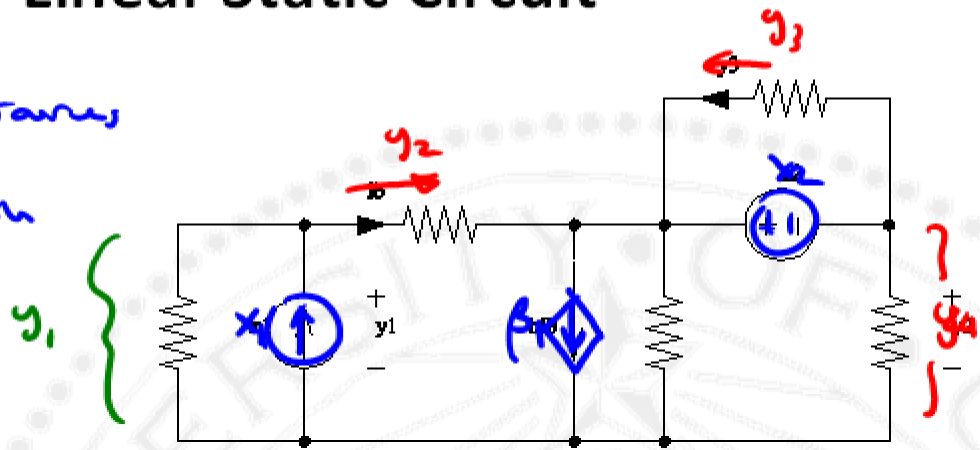
$y_4 \; y_5 \; y_6$ — torques

"pSPICE"

Linear Static Circuit

x_j – independent sources

y_i – circuit variables



$y = Ax$

A = "impedance matrix"

a_{11} — input impedance

a_{31} — current gain

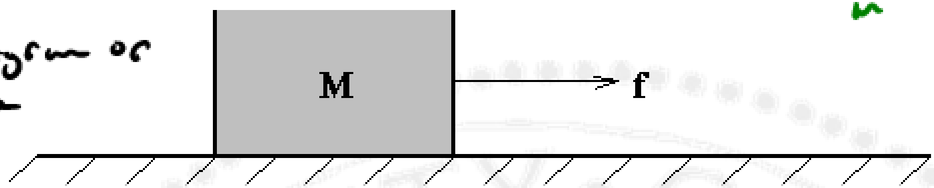
a_{41} — ohm

$f = ma \quad f = m\dot{v} \rightarrow v = \frac{1}{m}\int_{k}^{k+1} f(t)\,dt$

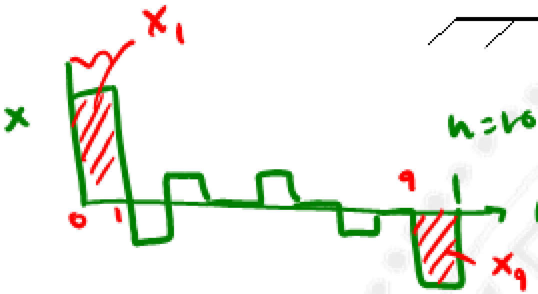
$v = \frac{f}{m}\,1$

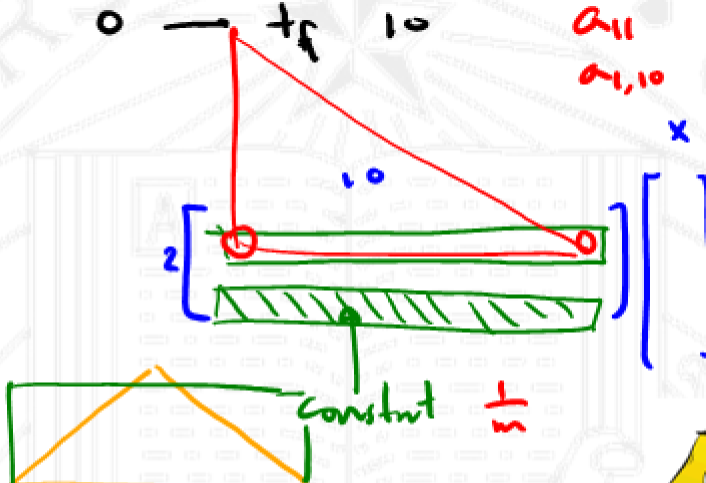
Final Position/Velocity of Mass

$x \in \mathbb{R}^n$ Some program or sequence

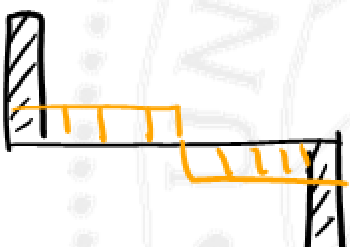


frictionless surface



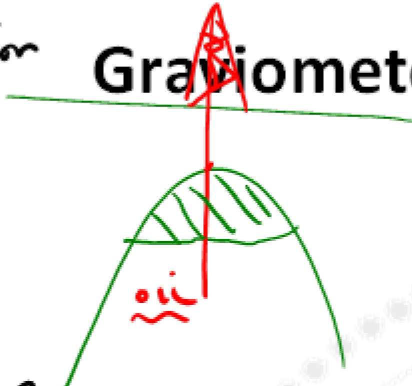


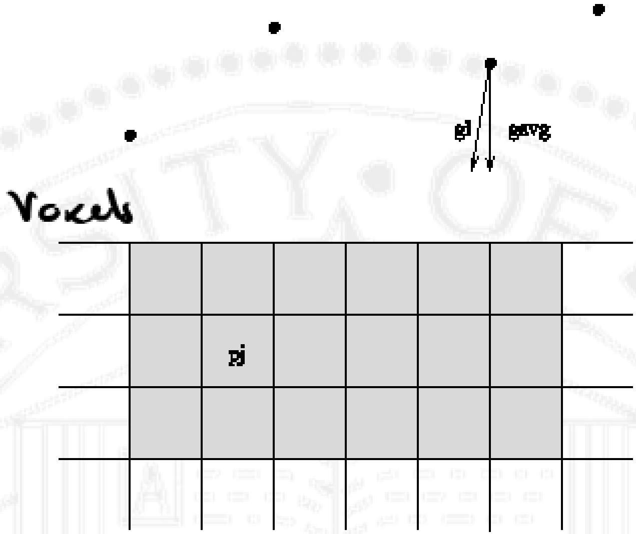
y output final position
final velocity



exquisite
parts/billion

Graviometer Prospecting





$x_j = \rho_j - \rho_{avg}$

y_i = measured gravitational

$g_i - g_{avg}$

$\underline{y = Ax}$

$x_j < 0$ salt water 1×10^{-7} g.

$x_j > 0$ submarine / metal deposit

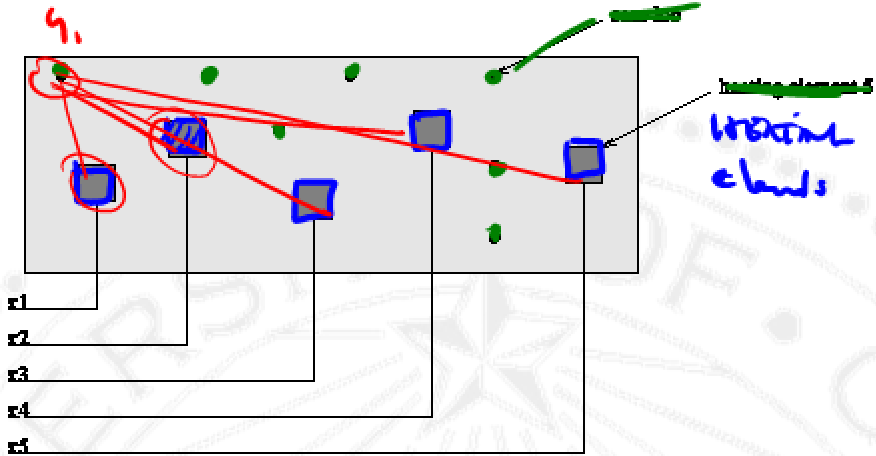
$x_j \ll 0$ gas.

Thermal System

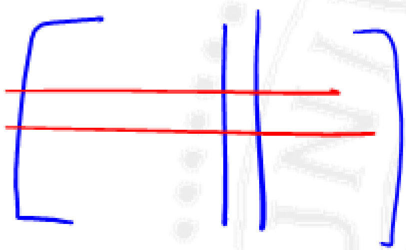
x_j power j^{th} heating element

y_i chg in steady state temp. @ location i

$y = Ax$



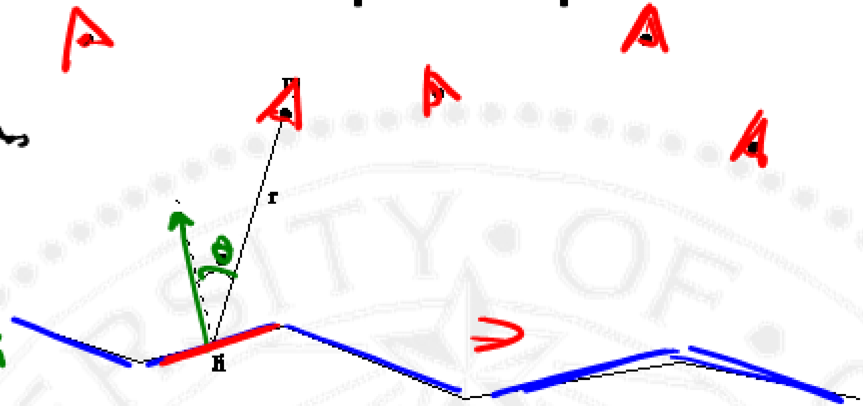
Diffusion / Poisson Eq'n



Illumination with Multiple Lamps

n lamps

m small patches no shadows



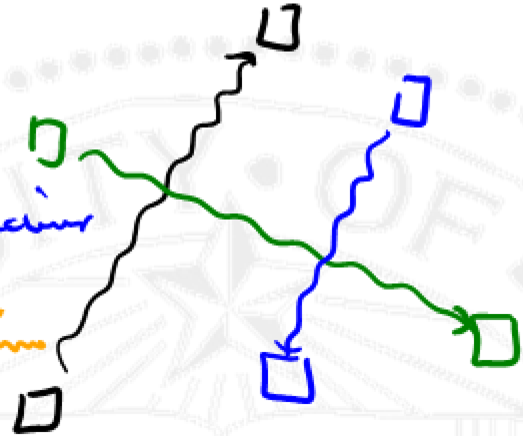
x_j power j^{th} lamp

y_i illumination on patch i

$$a_{ij} = r_{ij}^{-2} \max\{\cos\theta_{ij}, 0\}$$

Signal and Interference Power in Wireless System

n – transmitter/receiver pairs



p_j power in the j^{th} transmitter

s_i received signal power of i^{th} receiver

z_i interference power at i^{th} receiver

G_{ij} path gain from j to i

s_i / z_i = SNR

$s = Ap$

$$A_{ij} = \begin{cases} G_{ij} & i = j \\ 0 & \text{otherwise} \end{cases}$$

$z = Bp$

$$B_{ij} = \begin{cases} 0 & i = j \\ G_{ij} & i \neq j \end{cases}$$

Cost of Production (1.2)

$a_{25} < 4.$

x_j price per unit production input j

y_i cost per unit of product i

a_{ij} unit of production input required to manufacture one unit of product i

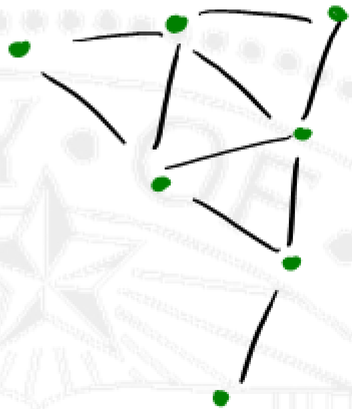
q_i quantity of product i $\quad y = Ax$

r_j total quantity of production input j that is needed.

$r = A^T q \qquad r^T x = (A^T q)^T x = q^T A x = q^T y$

Cost of Production (2.2)

n flows



Network Traffic and Flows (1.2)

n flows with rates $f_1 \dots f_n$

t_i traffic on link i

Σ flows that pass through link i

flow link incidence matrix

$A_{ij} = 1$ flow j goes over link i

$\phantom{A_{ij} =}\ 0$ otherwise

$t = Af$

↑

edges in network

$$A = \begin{bmatrix} & 1 & \\ & 1 & \\ & 0 & \\ & 1 & \end{bmatrix}$$

Network Traffic and Flows (2.2)

$d_1 \cdots d_m$ link delays

$l_1 \cdots l_n$ latency (total travel time) flows

$$l = A^T d$$

$$f^T l = f^T A^T d = (Af)^T d = t^T d$$

A^T has a very intuitive physical interpretation

Linearization

$f: \mathbb{R}^n \to \mathbb{R}^m$ differentiable @ $x_0 \in \mathbb{R}^n$ then:

if x near x_0 $\to$ $f(x)$ very near $f(x_0) + Df(x_0)(x - x_0)$

$$Df(x_0)_{ij} \triangleq \left.\frac{\partial f_i}{\partial x_j}\right|_{x = x_0} \leftarrow \text{"jacobian"}$$

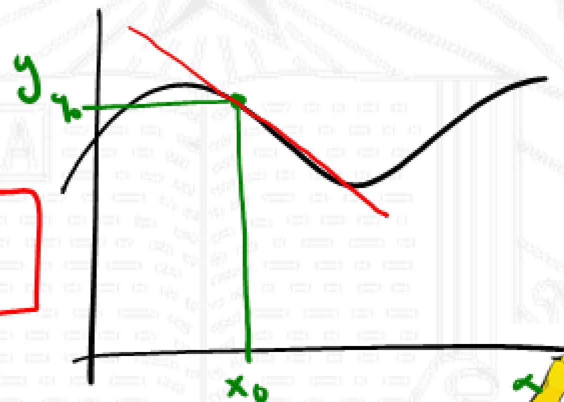
$y = f(x) \leftarrow$ non-linear

$y_0 = f(x_0)$

$\delta x \triangleq x - x_0$

$\delta y \triangleq y - y_0$

$$\boxed{\delta y \simeq Df(x_0)\,\delta x}$$

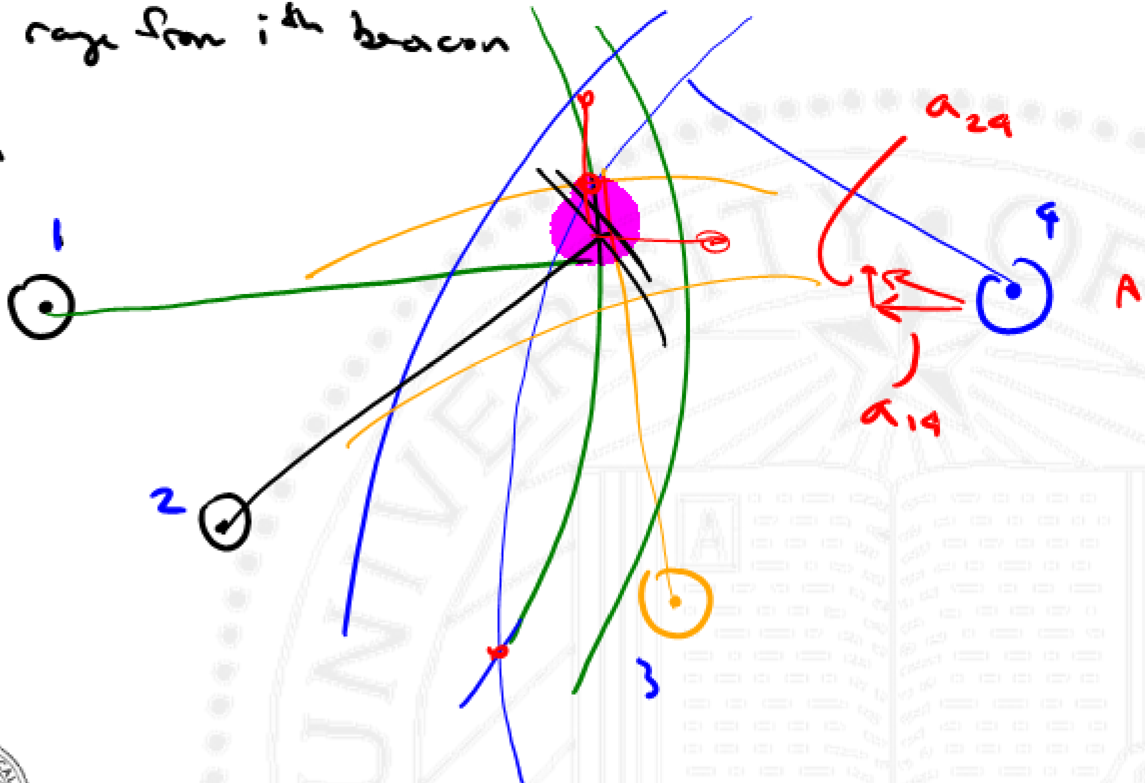


MULTI-LATERATION SYSTEM

Navigation by Ranging (1.2)

ρ_i range from i^{th} beacon

x_i, y_i



Navigation by Ranging (2.2)

$\rho \in \mathbb{R}^4$ is nonlinear function of $(x,y) \in \mathbb{R}^2$

$$\rho_i(x,y) = \sqrt{(x-p_i)^2 + (y-q_i)^2} \qquad \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

$$\delta\rho \cong A \begin{bmatrix} \delta x \\ \delta y \end{bmatrix}$$

$$a_{i1} = \frac{x_0 - p_i}{\sqrt{(x_0-p_i)^2 + (y_0-q_i)^2}} \qquad a_{i2} = \frac{y_0 - q_i}{\text{same.}}$$

$$A = \begin{bmatrix} \cdot & \cdot \\ \boxed{\cdot \quad \cdot} \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$$

Broad Categories of Applications

- Linear model or function y=Ax
- Some broad categories of applications:
  - Estimation or inversion
  - Control or design
  - Mapping or transformation

(this list is not exclusive; can have combinations)

Estimation or Inversion

y_i is the i^{th} measurement or sensor reading (which we know)

x_j parameter to be estimated or determined

a_{ij} sensitivity of i^{th} sensor to j^{th} parameter

→ find x, given y

→ find all x's consistent w/ y.

→ if no x exists such that $y = Ax$

find the best x that is the "most consistent" w/ y.

$y = Ax + v$ ← noise

Control or Design

$$y = Ax$$

x is a vector of design parameters or inputs we can choose

y is the output or result.

find x such that $y = y_{des}$.

find all x's that yield y_{des}

Among all x's that give me y_{des}

choose the best one

Mapping or Transformation

x is mapped or transformed by A into y.

determine if there is an $x \to y$.

find an x that maps onto y

find all x's that map onto y

decode or undo transformation.