Final Review

A(:, i) ← i^th column

A(j, :) ← j^th row

MATLAB:

orth( ) — orthogonal subspace basis vectors.

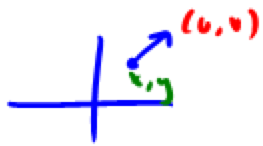
c2d(,W
'zoh'

qr( ) — QR decomposition

null ( ) — null space basis vector.

norm(x,i)
'inf'
'frob'

quiver(x,y,u,v)



svd(, 'econ') ⟩ Σ has no zeros.

A(:)
reshape

sys = ss(A,B,C,D ← sets up LDS
,Ts

step(sys)
impulse(sys)

lsim(sys, t, u).
dlsim(sys,t,u)

mod( )
rem( )

size(A) → (m,n)

rank(A) → dim(span(A)).

eig(A) → eigenvalues, eigenvectors

length(A) → max(m,n)

max( )   max(A(:))

min( )

find(A(:)==0)

sort(X) →

th = (th<-180)*360 - (th>180)*360 + th;

(a>b)

$\begin{bmatrix}1\\1\\0\\1\end{bmatrix}$

[1 3 0 4]

roots(——) $x^3 + 3x^2 + 0x + 4$

conv(A, B) convolution

polyval([...], x) —

> lookfor

PARTING THOUGHTS

→ LINEAR ALGEBRA — NOW WE ACTUALLY SOLVE PROBLEMS

10-20 years ago, "talked" about it..

- MATLAB – great for poking around.
- LAPACK — 500-1000 variables "trivial"

  much higher structure exploit.

  Sparse — 100k variables ~ 10 sec

  1M x 1M doable.

→ Levels of Understanding

Simple, Intuitive view: (high school math)

17 var 17 eq'ns — can solve.

80 var 60 eq'ns — 20 degrees of freedom

Platonic View (Advanced undergrad).

singular, rank, range, nullspace, Jordan Form, controllability, observability.

Precise, unambiguous

Insight deeper understanding

Qualitative View: (Grad. level)

- least squares, SVD
- Numerical measures on ideas Singular
- Interpretations depend on context.
- Very useful, useful in practice.

Remember which level you are operating on!!

(x, y, Val)

- $\dot{x} = Ax$ AUTONOMOUS SYSTEMS ($u = \phi$)

- i/o $\dot{x} = Ax + Bu$

  $y = Cx + Du$

- Cayley-Hamilton (C-H).

- Matrix Exponential / Laplace Transform

- Jordan Form

- Eigenvalues / Eigenvectors / Diagonalization

- Symmetric Matrices, Matrix Norm, Quadratic Forms

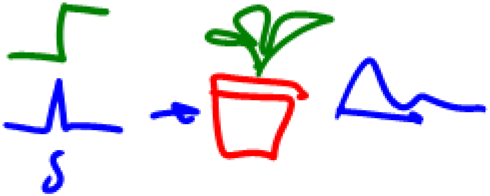
  $x^T A x$.

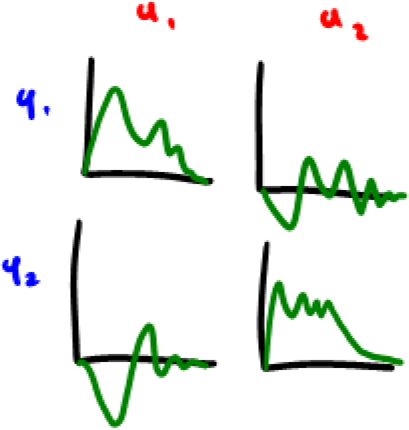
- SVD, SVD Applications (Ellipsoidal bounds)

- Controllability / Observability.

Step response

Impulse response



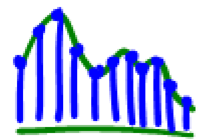


$$H(s) = C(sI - A)^{-1}B + D \, .$$

$$\frac{H(s)}{s}$$

$$\mathcal{L}\{\delta(t)\} = 1 \, .$$

$$\underline{Y(s) = H(s)U(s) \, .}$$



$$y(t) = \mathcal{L}^{-1}\{H(s)U(s)\}$$

$\mathcal{L}\{$  $\}$

$$\frac{1}{s}$$

$$A \geq 0 \quad \longleftrightarrow \quad x^T A x \geq 0 \quad \forall\, x.$$

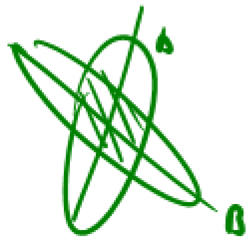
square,

$$\left(\frac{A+A^T}{2}\right)$$

$$A > 0 \quad - \text{ inverse exists.}$$

$$A > B \qquad x^T A x > x^T B x \quad \forall\, x.$$

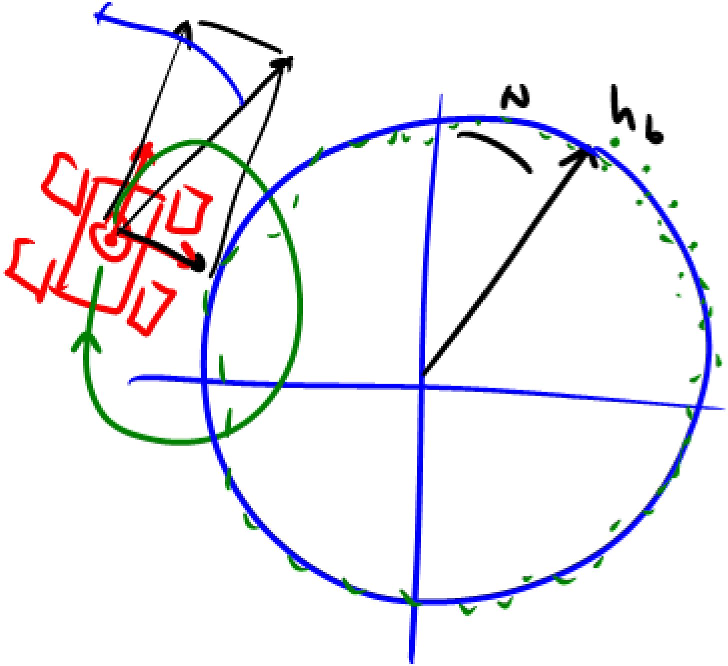
$$\left.\begin{array}{l} A \not> B \\ B \not> A \end{array}\right\} \text{possible}$$



Rules For Final

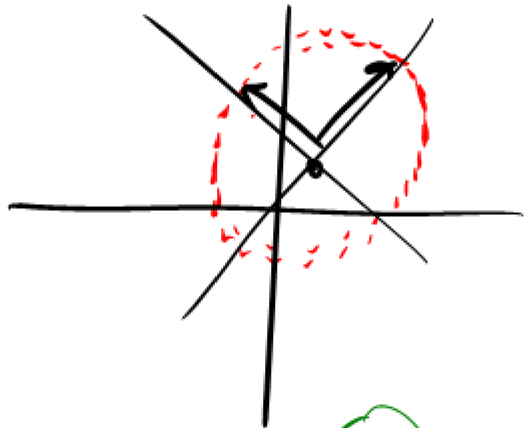
1. Absolutely no talking about it to ANYONE!! (except Gabe).
2. Under pain of death, see 1.
3. Open notes, open books, open videos, open wikipedia, open Google.

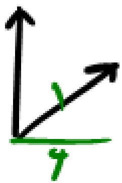
Except: published solutions.


ω
h_b

$$x = \frac{h\cos\psi}{a} + x_0$$

$$y = \frac{h\sin\psi}{b} + y_0$$





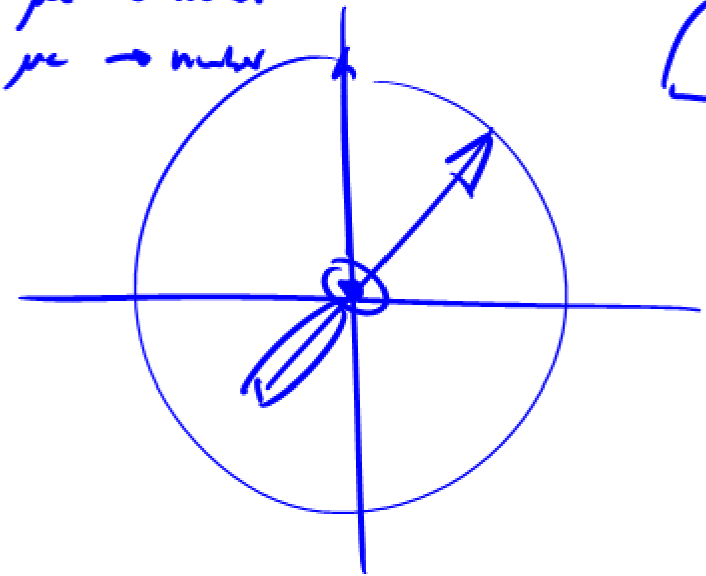
$$y = A\begin{bmatrix} x \end{bmatrix}$$

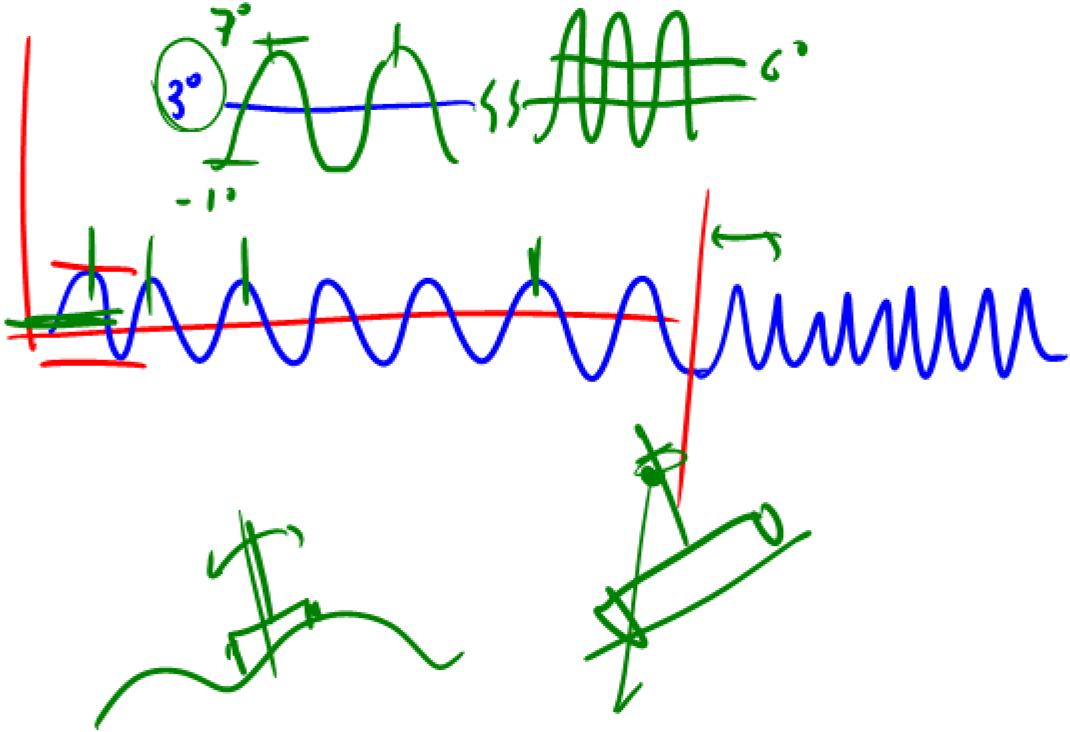
$$\frac{x_0}{y_0}$$

$$a x_0$$

$$\begin{bmatrix} y \\ \vdots \\ y \end{bmatrix} = \begin{bmatrix} x & x^2 & xy & y^2 & 1 \\ | & | & | & | & | \end{bmatrix} \begin{bmatrix} \vdots \end{bmatrix}$$

cos θ — μc → motor
sin θ — μc → motor



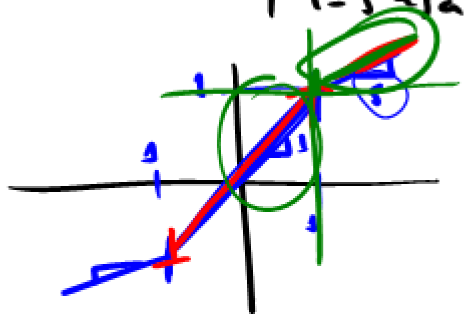

7°
3°
6°
-1°

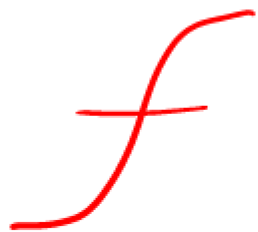
6

$$u \longrightarrow \boxed{\phi} \xrightarrow{v} \boxed{*h} \longrightarrow z$$

$$z(t) = \sum_{\tau=0}^{M-1} h(\tau)\, v(t-\tau) \qquad v(t) = \phi(u(t)).$$

$$\phi : \mathbb{R} \to \mathbb{R} \qquad \phi(a) = \begin{cases} -1+s+sa & a<1 \\ a & -1 \le a \le 1 \\ 1-s+sa & a>1 \end{cases}$$





$$z \rightarrow \boxed{*g} \xrightarrow{\hat{v}} \boxed{\psi} \rightarrow \hat{u}$$

$$u \rightarrow \underbrace{\boxed{\phi} \rightarrow \boxed{*h} \rightarrow \boxed{*g} \rightarrow \boxed{\psi}}_{I\,.} \rightarrow \hat{u}$$

$$\hat{v}(t) = \sum_{\tau=0}^{M-1} g(\tau)\, z(t-\tau) \qquad \hat{u} = \psi(\hat{v}(t)) \qquad t \in \mathbb{Z}$$

$$\begin{bmatrix} g * h \approx \delta \\ \psi \approx \phi^{-1} \end{bmatrix}$$

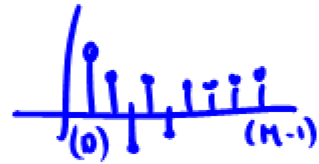
$$(g*h)(t) = \sum_{\tau=\max(0,\,t-M+1)}^{\min\{M-1,\,t\}} g(\tau)\, h(t-\tau)$$

$u(1) \ldots u(N)$, $z(1) \ldots z(N)$, M.

$\hat{s}$, $g(0) \cdots g(n-1)$

$$\min J = \frac{1}{N-n+1} \sum_{i=n}^{N} \left(\hat{r}(i) - \underline{\phi(u(i))}\right)^2$$

if $(g*h) = \delta$ $\quad \hat{s} = s \rightarrow J = 0$.



$$J = \frac{1}{(N-n+1)} \| y - A x \|^2$$

$$x = \begin{bmatrix} g(n-1) \\ \vdots \\ g(0) \\ s \end{bmatrix}$$

$$y_i = \begin{cases} 1 & u(i+n-1) > 1 \\ -1 & u(i+n-1) < -1 \\ u(i+n-1) & \text{otherwise} \end{cases}$$

$$(u - [Z]x)$$