Observability and State Estimation

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Observability and State Estimation

- State estimation
- Discrete-time observability
- Observability-Controllability duality
- Observers for noiseless case

. LEAST SQUARES OBSERVERS (KALMAN FILTER)

State Estimation Set-Up (1.2)

$x_{k+1} = \Phi x_k + \Gamma u_k + w_k$

↑ process noise

$y_k = H x_k + D u_k + v_k$

← measurement noise

w_k ← state disturbance or process noise

v_k ← measurement noise or error

Φ	Γ
H	D

known

u_k observed

y_k $[0, k-1]$

State Estimation Set-Up (2.2)

given $u_0 \cdots u_{k-1}$ $y_0 \cdots y_{k-1}$ estimate x_k.

$k=0$ initial state

$k-1$ current state

k predicted next state

x_{k-5} ← smoother

x_{k+10} ← predictor

$\hat{x}_k$ observer estimator

$\hat{x}(s|t-1)$

State Estimation Problem (1.2)

State Estimation Problem (2.2)

Noiseless case (1.3)

$x(0) = 0 \quad w_k = 0 \quad v_k = 0$

$x_{k+1} = \Phi x_k + \Gamma u_k \qquad y_k = H x_k + D u_k$

$x_k \in \mathbb{R}^n$

$y_k \in \mathbb{R}^p$

$u_k \in \mathbb{R}^m$

$$\begin{bmatrix} y_0 \\ \vdots \\ y_{k-1} \end{bmatrix} = \mathcal{O}_k x_0 + \mathcal{J}_k \begin{bmatrix} u_0 \\ \vdots \\ u_{k-1} \end{bmatrix}$$

Noiseless case (2.3)

$$\mathcal{O}_k = \begin{bmatrix} H \\ H\Phi \\ H\Phi^2 \\ \vdots \\ H\Phi^{k-1} \end{bmatrix} \qquad \mathcal{J}_k = \begin{bmatrix} D & 0 & \cdots & 0 \\ H\Gamma & D & 0 & \cdots 0 \\ H\Phi\Gamma & H\Gamma & D & 0 \cdots \\ \vdots & & & \\ H\Phi^{k-2}\Gamma & \cdots & & \end{bmatrix}$$

↑ skinny

$$N(\mathcal{O}_k) = \{0\}.$$

Noiseless case (3.3)

$$\mathcal{O} = \mathcal{O}_n = \begin{bmatrix} H \\ H\Phi \\ \vdots \\ H\Phi^{n-1} \end{bmatrix}$$ using c-h.

$N(\mathcal{O})$ is unobservable subspace

if $\mathcal{O}$ is full rank — $\text{rank}(\mathcal{O}) = n$

reconstruct x from inputs & measurements

Observability Matrix (1.2)

Observability Matrix (2.2)

Observability – Controllability Duality

Observability – Controllability Duality

$$\left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right] \rightarrow \left[\begin{array}{c|c} \tilde{A} & \tilde{B} \\ \hline \tilde{C} & \tilde{D} \end{array}\right] = \left[\begin{array}{c|c} A^T & C^T \\ \hline B^T & D^T \end{array}\right]$$

$$\tilde{\mathcal{C}} = \left[\tilde{B} \quad \tilde{A}\tilde{B} \quad \tilde{A}^2\tilde{B} \cdots\right]$$

$$= \left[C^T \quad A^T C^T \cdots (A^T)^{n-1} C^T\right] = \mathcal{O}^T$$

$$\tilde{\mathcal{O}} = \mathcal{C}^T$$

Observers for Noiseless Case (1.2)

Find: 5 problems

- No human help
- Take 24 hours
- No published solutions

Send email to me & shroom when you start.

Observers for Noiseless Case (2.2)

Invariance of Unobservable Set (1.2)

Invariance of Unobservable Set (2.2)

Continuous-Time Observability (1.3)

Continuous-Time Observability (2.3)

Continuous-Time Observability (3.3)

A converse (1.2)

A converse (2.2)

Least-Squares Observers (1.2)

Least-Squares Observers (2.2)

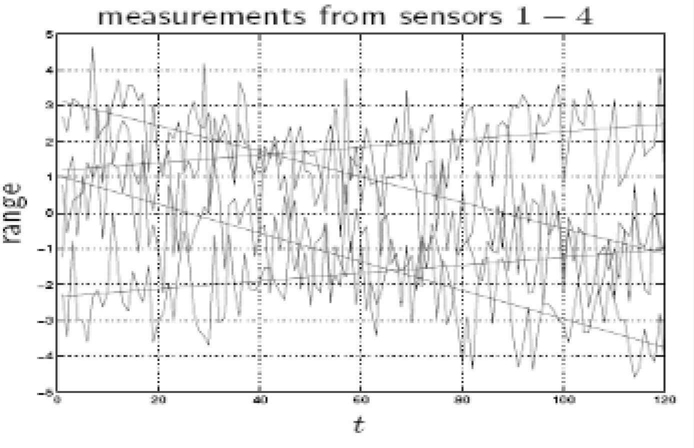
Least-Squares Observer Uncertainty Ellipsoid (1.2)

Least-Squares Observer Uncertainty Ellipsoid (2.2)

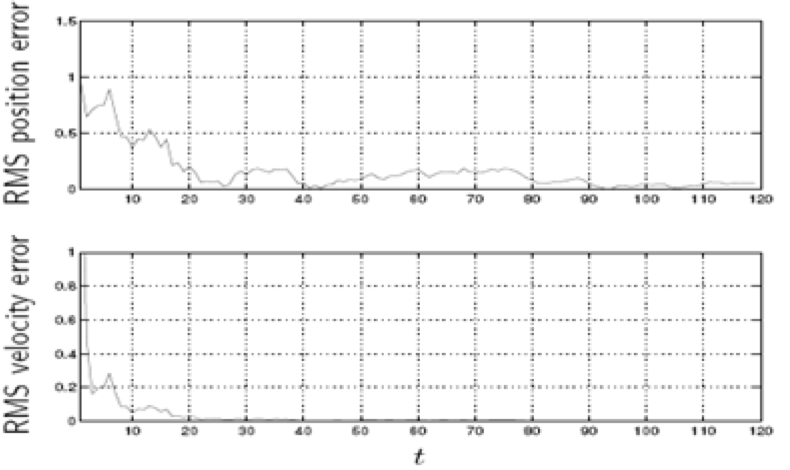
Infinite Horizon Uncertainty Ellipsoid (1.2)

Infinite Horizon Uncertainty Ellipsoid (2.2)

Example: Particle Motion (1.2)



Example: Particle Motion (2.2)



Continuous-Time Least Squares State Estimation (1.3)

Continuous-Time Least Squares State Estimation (2.3)

Continuous-Time Least Squares State Estimation (3.3)

Questions?