Homework 5

February 9, 2018

1 Problem 1: Gravity waves in a linearly stratified medium

Question 1: What are the ray paths of gravity wave packets in an ocean basin of uniform depth H that has $N = N_0$ (constant), where $Z = 0$ is the surface of the water, and Z increases downward? Note that you may assume that the basin is infinite in the horizontal direction. Plot an upward moving ray path starting from $X = 0$ and $Z = H$.

Question 2: Answer the same question, but with $N(Z) = aZ$.

2 Problem 2: Global modes

Consider again the same ocean basin, again with constant $N = N_0$, and of finite horizontal extent L. Assume that the vertical velocity w is zero both at the surface of the water, and at $Z = H$ (the bottom of the basin). Assume that the horizontal velocity is zero on the side-walls of the basin. Find the *global modes* of this system. Discuss their spatial properties, and contrast their discrete dispersion relation with the one derived from assuming monochromatic plane wave solutions.

3 Problem 3: Surface waves near the beach

Question 1. Using only the dispersion relation for surface waves in the limit where surface tension is negligible ($\sigma = 0$), find out what the evolution equations for ω and $\boldsymbol{k} = (k_x, k_y)$ of a wave packet $\eta(x, y, t) = A(X, Y, T)e^{i\theta}$ would be.

Question 2. Consider a simple model in the (x, y) plane where there is a beach along the line $y = 0$, and the ocean is in the half-plane $y > 0$. The ocean floor away from the beach has a sloping bottom $h(x, y) = sy$ where s is the slope. A wave is coming towards the beach from far away, and is approaching it at an angle. Explain, using ray path theory, why the wave crests become parallel to the beach by the time the wave arrives at $y = 0$.