Homework 2 (Chapter 2 part 1)

January 17, 2018

1 Sound waves with gravity (part 1)

Question 1: Consider an isothermal atmosphere in a Cartesian coordinate system (see Chapter 1). Show that the equation for isothermal sound waves in that isothermal atmosphere, without neglecting gravity, is

$$\frac{\partial^2 \tilde{\rho}}{\partial t^2} = c^2 \nabla^2 \tilde{\rho} + g \frac{\partial \tilde{\rho}}{\partial z} \tag{1}$$

Question 2: Assume 1D monochromatic plane wave solutions to the wave equation derived above, assuming that $\tilde{\rho}$ only varies with z (i.e. ignore x and y dependences to only study $\tilde{\rho}(z,t)$)

- What is the dispersion relation?
- Based on the dispersion relation only, under which circumstances is the term that includes gravity negligible?
- Plug in typical dimensional numbers for g, c, k, etc.. Is gravity typically negligible for isothermal sound waves in air? Are there any circumstances in which it may not be negligible?

2 d'Alembert's solution

Solve the 1D Cartesian wave equation $\partial_{tt} p = c^2 \partial_{xx} p$ subject to initial conditions $p(x,0) = p_0 \exp(-x^2/2)$ and $\partial_t p(x,0) = 1$. How does this solution differ from the case where $\partial_t p(x,0) = 0$ studied in class? Sketch the wave packet evolution as a function of time to illustrate your argument (alternatively, plot it on the computer for $p_0 = 1$, $c = 1$, at carefully selected times, for instance).

3 Superposition of monochromatic waves vs. d'Alembert's solution

Solve the 1D Cartesian wave equation $\partial_{tt} p = c^2 \partial_{xx} p$ subject to initial conditions $p(x,0) = p_0 \exp(-x^2/2)$ and $\partial_t p(x,0) = 0$ using a superposition of monochromatic waves. How does this relate to d'Alembert's solution for the same initial conditions? Hint: I find that it helps to remember that, for any complex number, $\Re(z) = (z + z^*)/2$.

4 Other coordinate systems / other equations ?

Question 1: Does d'Alembert's solution method, or a similar one, work for 1D problems in polar coordinates or in spherical polar coordinates, i.e. for the following wave equations:

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) \text{ in polar coordinates}$$
$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) \text{ in spherical polar coordinates} \tag{2}$$

If yes, explain the method. If no, explain why it doesn't work.

Question 2: Does d'Alembert's solution method, or a similar one, work for the equation derived in Problem 1 (waves with gravity?)