Homework 5

February 17, 2016

Do problem 1, and then either problem 2 or problem 3.

1 Local analysis for convection

Redo the local stability analysis we did in class, but this time add the dissipation terms we neglected: the viscous dissipation term in the momentum equation, and the thermal dissipation term in the temperature equation.

What changes and what stays the same? Is λ still independent of $|k|$? What do the fastest-growing modes look like? You may assume that the system is 2D. It is not necessary, but will make your calculation easier.

2 Rayleigh Benard convection with Dedalus (option 1)

1. Modify the 2D Rayleigh Benard script to study stress-free boundary conditions. Let the total horizontal domain size be $L_x = 5$, and the vertical one be $L_z = 1$

2. Set the Prandtl number to be 1. First, test that for Rayleigh number is 650 (just below critical), the system is stable. Present one plot that supports this conclusion (e.g. total energy in the system, or total flux for instance).

3. Then do a run for Rayleigh number 700. What is the ultimate behavior of the system (e.g. steady-state, time-periodic, quasi-periodic, chaotic, etc.). Present a plot to support your conclusion (e.g. total energy in the system, or total flux for instance). Does the convective mode that appear satisfy your results from linear stability analysis, both in terms of its growth rate, and in terms of its spatial structure? Present plots to support your argument.

4. Run a number of simulations for increasing Ra, and each time, measure the total heat flux (i.e. for instance, the temperature derivative at the top boundary). Plot that heat flux against Ra. What do you find?

Hints: think about your resolution carefully – for questions 1, 2 and 3, the simulations are not very turbulent, so no need to have too much resolution. Also think about your timestep, what you can get away with.

3 Rayleigh Benard convection with rotation (option 2)

1. What is the criterion for linear instability in the presence of rotation, when the rotation axis is in the same direction as gravity? (see most books on hydrodynamic stability for help). Hint: you will need to it in 3D this time.

2. What happens when one tried to do energy stability for Rayleigh-Benard convection with rotation? (Note: you don't have to do it all the way, the answer to this question lies in the very first steps of the calculation.).