Homework 2

January 11, 2016

1 Problem 1. Superposition of monochromatic waves

Solve the 1D Cartesian wave equation $\partial_{tt}p = c^2\partial_{xx}p$ subject to initial conditions $p(x,0) = p_0\exp(-x^2/2)$ and $\partial_t p(x,0) = 0$ using a superposition of monochromatic waves. Relate this solution to d'Alembert's solution for the same initial conditions. Hint: I find that it helps to remember that, for any complex number, $\Re(z) = (z + z^*)/2$.

2 Problem 2. Global modes in a square, global modes in a disk

Find the 2D eigenmodes and eigenvalues of the wave equation $\frac{\partial^2 p}{\partial t^2} = c^2\nabla^2 p$

- in a square whose side length is 1
- in a disk of radius 1.

subject to $p = 0$ on the boundary of the region, and some general regularity conditions. Plot a few representative eigenmodes in each case.

3 Problem 3. Wave packet approximation

Using the wave packet approximation, find the solution to the following 2D problem in the infinite Cartesian plane:

$$\frac{\partial^2 p}{\partial t^2} = c^2\nabla^2 p \tag{1}$$

where $p(x,y,0) = p_0\cos(x+y)\exp\left(-\frac{x^2+y^2}{200}\right)$ and every other quantity is otherwise 0. A script of the Dedalus code to solve this equation will be provided. (Hint: it helps to first see the solution.)

Compare qualitatively your analytical solution with the numerical one. Discuss the differences.

4 Problem 4: A sound mirror

At the San Francisco Exploratorium, there is a rather amazing example of a sound mirror. The display is composed of two walls facing each other at two ends of a (usually very crowded) room, and each wall has a 2m-size half-sphere carved into it. When one person stands at the edge of the half-sphere and speaks into it, even very gently, another person standing in the same position but near the other half-sphere can hear the first person's voice very clearly. If, on the other hand, the first person turns around and speaks directly towards the second, it's usually impossible for the two people to hear each other across the noisy room.

Explain how the sound mirror works, and why speaking directly to one another wouldn't. Hint: use ray theory and assume that a half-sphere is not too different from a parabola for simplicity. A parabola has the property that parallel rays coming in from infinity all focus in the same point (called the focal point). Similarly, rays emitted from the focus end up going to infinity on parallel lines, after reflection on the mirror. See Wikipedia article on Parabolic Reflectors for instance.

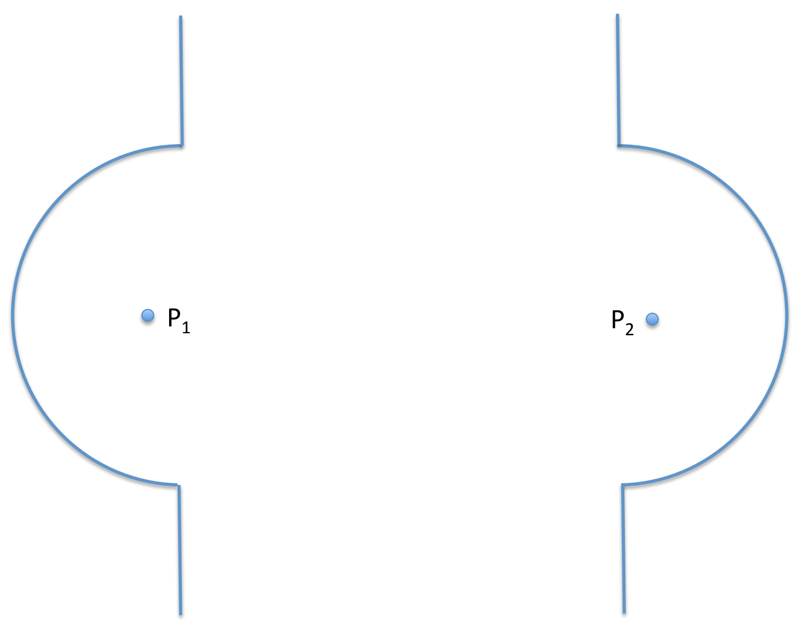

Figure 1: Schematic of the sound mirror. The two people are sitting at the two foci of the mirrors at P_1 and P_2, talking to each other while facing the mirrors, can hear each other across a crowded, noisy room.