# Homework 1

January 5, 2016

# 1 Chapter 1 problems.

## 1.1 Problem 1: Hydrostatic equilibrium of an isothermal atmosphere

Hydrostatic equilibrium is an equation describing the state of a fluid with no fluid motion, when there is an exact balance between the pressure gradient and the gravitational force, namely

$$-\nabla p + \rho \boldsymbol{g} = 0 \tag{1}$$

**Question 1:** Express this equation in a Cartesian coordinate system, assuming that the gravity $\boldsymbol{g} = -g\boldsymbol{e}_z$.

**Question 2:** Assume a perfect gas equation of state. Express hydrostatic equilibrium as an equation for the density only. Solve this equation from $z = 0$ (the ground) upward, assuming that $\rho = \rho_0$ at the ground. Interpret your solution.

## 1.2 Problem 2: Mass conservation in polar coordinates.

**Question 1:** Consider the mass conservation equation, and express it in 2D polar coordinate $(r, \theta)$.

**Question 2:** Assume that the velocity field $\boldsymbol{u} = (u_r, u_\theta) = (u_0, 0)$, where $u_0$ is constant. Sketch this velocity field.

**Question 3:** Solve the mass conservation equation, assuming that $\rho(r, 0) = r\exp(-(r-1)^2/2)$. Plot the results, and discuss your findings. Hint: mathematically, it may be easier to consider the function $f(r) = r\rho(r)$.

# 2 Chapter 2:

## 2.1 Problem 1: Sound waves with gravity (part 1)

Consider an isothermal atmosphere in a Cartesian coordinate system (see Problem 1 of Chapter 1). Show that the equation for isothermal sound waves in that isothermal atmosphere, this time without neglecting gravity, is

$$\frac{\partial^2 \tilde{\rho}}{\partial t^2} = c^2 \nabla^2 \tilde{\rho} + g \frac{\partial \tilde{\rho}}{\partial z} \tag{2}$$

## 2.2 Problem 2: Sound waves with gravity (part 2)

Assume 1D monochromatic plane wave solutions to the wave equation derived above, assuming that $\tilde{\rho}$ only varies with $z$.

- What is the dispersion relation?
- Based on the dispersion relation only, under which circumstances is the term that includes gravity negligible?
- Plug in typical dimensional numbers for $g$, $c$, $k$, etc.. Is gravity typically negligible for isothermal sound waves in air? Are there any circumstances in which it may not be negligible?

## 2.3 Problem 3: d'Alembert's solution

Does d'Alembert's solution method, or a similar one, work for 1D problems in polar coordinates or in spherical polar coordinates, i.e. for the following wave equations:

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial f}{\partial r} \right) \text{ in polar coordinates}$$

$$\frac{\partial^2 f}{\partial t^2} = c^2 \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial f}{\partial r} \right) \text{ in spherical polar coordinates} \tag{3}$$

If yes, explain the method. If no, explain why it doesn't work.