

Homework 5

Problem 1: Consider the problem

$$\begin{aligned}\epsilon f'' + f' + f &= 0 \text{ where } \epsilon < 0 \\ f(0) &= e \\ f(1) &= 0\end{aligned}\tag{1}$$

- Find an exact solution of the problem, and plot it for $\epsilon = -0.1$. Where is the boundary layer, and what is its thickness?
- Find the 1-term expansion of this solution using a multi-scale analysis.
- Find the lowest-order composite expansion of this problem using boundary layer analysis.
- Plot the solutions to the last 2 questions on the same plot as the exact solution. Discuss your findings.

Problem 2: Consider the problem

$$\begin{aligned}\epsilon f'' + f' + e^f &= 0 \\ f(0) &= 0 \\ f(1) &= 1\end{aligned}\tag{2}$$

for $\epsilon > 0$.

- Using any method of your choice (numerical solution, method of dominant balance, trial and error, ...) determine where the boundary layer is, and what its thickness is.
- Find the lowest-order composite expansion of this problem using boundary layer analysis.
- Plot the solution for various values of ϵ . Discuss your findings.