Homework 1

Partial Differentiation:

- RHB 5.1, 5.5, 5.8
- If $xyz + x^3 + y^4 + z^5 = 0$ find $(\partial x/\partial y)_z$, $(\partial y/\partial z)_x$, $(\partial z/\partial x)_y$, and verify that their product is -1.
- The function $f(x, y)$ is a scalar function of position on the $x-y$ plane. Position can also be specified by the coordinates u and v which are relative to axes rotated by an angle θ from the x and y axes (note that θ is constant). Show that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}$$

Vector Calculus:

- A vector field is $\mathbf{F}(\mathbf{x}) = (x^2 - 3x + \ln(z), 2x^4 + e^y, \sin(xz))$. Calculate $\nabla \cdot \mathbf{F}$ and $\nabla \times \mathbf{F}$.
- Evaluate the gradient of $f(x, y, z) = \frac{zx^2}{x^2+y^2+z^2}$.
- RHB 10.13, 10.14, 10.15
- If $\mathbf{r}$ is the position vector (e.g. in Cartesian coordinates, $\mathbf{r} = (x, y, z)$), what is $\nabla \cdot \mathbf{r}$ and $\nabla \times \mathbf{r}$ in Cartesian, cylindrical and spherical coordinate systems? What is $\nabla \cdot \big(\mathbf{r}/|\mathbf{r}|^3\big)$? What is $\nabla \times \big(\mathbf{r}/|\mathbf{r}|^3\big)$?

First order ODEs:

- RHB 14.2, 14.5, 14.6
- RHB 14.11, 14.16, 14.24
