Algebra Workshop 3: Rational expressions

1 Simplfying rational expressions by factoring

RULE:

- Rational expressions in expanded form cannot be simplified directly. To simplify a rational expression, it must first be factored. Then, if factors cancel out, then the expression can be simplified.
- Remember that
$$\frac{a-b}{b-a} = -1$$
- Remember that for any expression E (which could be a function of x)
$$\frac{E^n}{E^m} = E^{n-m}$$

EXAMPLE 1
$$\frac{x^2-4}{(x-2)^2} = \frac{(x-2)(x+2)}{(x-2)^2} = \frac{x+2}{x-2}$$
provided $x \neq 2$. Remember that you cannot simplify x^2 and x, or 4 and 2 in the original expression. That would not make mathematical sense.

EXAMPLE 2
$$\frac{3-x}{x^2-x-6} = \frac{3-x}{(x-3)(x+2)} = -\frac{1}{x+2}$$

PRACTICE: Simplify the following expressions as much as possible.

- $\frac{2-x}{x(x-2)}$
- $\frac{2x+2}{2x-6}$
- $\frac{5x+30}{x^2+6x}$
- $\frac{x^2-x-12}{16-x^2}$
- $\frac{5x-2}{4-25x^2}$
- $\frac{x^3+4x^2-3x-12}{(x+4)^2}$

2 Multiplying rational expressions the efficient way

When multiplying rational expressions, it is tempting to just multiply out the respective numerators and denominators. However, in some cases, it pays off to first factor expressions, and see if there are cancellations before expanding things out...

EXAMPLE 1:
$$\frac{x^2}{x+3} \cdot \frac{x+5}{x-7} = \frac{x^2(x+5)}{(x+3)(x-7)} = \frac{x^3+5x^2}{x^2-4x-21}$$

if we need the expression in expanded form. In this example, there was no obvious cancellation in the factored form, so we can indeed just multiply out the factors. But compare with

EXAMPLE 2:

$$\frac{x+3}{x-4} \cdot \frac{x^2-2x-8}{x^2-9} = \frac{(x+3)(x^2-2x-8)}{(x-4)(x^2-9)} = \frac{(x+3)(x-4)(x+2)}{(x-4)(x+3)(x-3)} = \frac{x+2}{x-3}$$

If instead we had multiplied out the factors in step 2, we would never have found this simple expression...

PRACTICE:

- $\frac{x-3}{x+7} \cdot \frac{3x+21}{2x-6}$
- $\frac{x^2-49}{x^2-4x-21} \cdot \frac{x+3}{x}$
- $\frac{6y+2}{y^2-1} \cdot \frac{1-y}{3y^2+y}$
- $(x+1) \cdot \frac{x+2}{x^2+7x+6}$
- $\frac{5x+5}{7x-7x^2} \cdot \frac{2x^2+x-3}{4x^2-9}$

3 Divisions of rational expressions by one another

Remember the one rule to simplify compound rational expressions (compound fractions): Flip them up and Multiply. Then, just use the method learned in the previous section.

EXAMPLE 1:

$$\frac{4x^2-25}{\frac{2x+5}{14}} = \frac{4x^2-25}{1} \cdot \frac{14}{2x+5} = \frac{14(4x^2-25)}{2x+5} = \frac{14(2x+5)(2x-5)}{2x+5} = 14(2x-5)$$

EXAMPLE 2:

$$\frac{\frac{x^2+3x-10}{2x}}{\frac{x^2-5x+6}{x^2-3x}} = \frac{x^2+3x-10}{2x} \cdot \frac{x^2-3x}{x^2-5x+6} = \frac{(x^2+3x-10)(x^2-3x)}{2x(x^2-5x+6)} = \frac{(x+5)(x-2)x(x-3)}{2x(x-3)(x-2)} = \frac{x+5}{2}$$

PRACTICE:

- $\frac{\frac{x+5}{7}}{\frac{4x+20}{9}}$
- $\frac{\frac{7}{y-5}}{\frac{28}{3y-15}}$
- $\frac{\frac{y^2+y}{y^2-4}}{\frac{y^2-1}{y^2+5y+6}}$
- $\frac{x^2+4x-5}{\frac{x^2-25}{x+7}}$