Handout 2: Final Review Worksheet

This worksheet will help you prepare for the Final. Note that all of the material required for the midterm (see previous review sheet) is also examinable for the Final. So, to prepare for the final, make sure you know all the material on both checklists.

Studying tips: Read each section. If you think you master the material, briefly look over the sample problems to convince yourself you know how to do them (if you're not sure how to do it, then try, and check your answers in the back,otherwise, quickly move on to the next section). If you don't think you master the material, go through the textbook and lecture notes about the material, make sure you understand the examples. Then do as many sample problems as you can, checking your answer in the back. The bottom line is: don't spend too much time on the points you know - make sure you spend most of your time studying for the points you're less familiar with.

If you have any difficulties, write them down, come to office hours to get help. Also, prepare some questions for the review on the last weekend.

The final will include *at least* 90% of questions which are extracted from the sample problems given in the review worksheets or from the Homework or from the Quizes.

1 Rational functions

See section 4.7. You need to know

- What the numerator and denominator are.
- How to reduce an expression to a rational function if necessary (see Handout 1)
- How to find the domain of definition of a rational function (i.e. exclude points where the denominator is 0).
- How to determine the behavior of the graph of the rational function for very large x (i.e. to look at the leading order terms in the numerator and in the denominator).
- What are, and how to find vertical, horizontal and oblique asymptotes.
- How to recognize that a rational function is fully factored or not
- How to factor it, if it is not fully factored (see Handout 1), and how to simplify it.
- What to do with "excluded points", points where both denominator and numerator have a root (i.e. to draw a circle where the point should be)
- How to draw a signs table for the rational function (remember the asymptotes).
- How to deduce from the signs table what the shape of the graph is. In particular, how to deduce what the shape of the graph near the asymptotes are.

Sample problems: pages 313-314 problems 9-34 (including, find the behavior near $+\infty$ and $-\infty$, and draw a signs table.)

2 Applied problems

The midterm will include one applied problem, similar to the ones given in the Homeworks and/or the problems discussed in class and in section. Remember that to solve applied problems:

- Identify what variables describe the problem. Give them names..
- Identify *all* the clues which relate the variables to one another. Write them as equations.
- Usually, you can then solve for one of the variables in one clue, then use that solution in another clue, or in what you are looking for, in order to make progress.

Solving word-problem is detective work! You need to use everything you know, and be creative!

Sample problems: page 269 problems 34-38, 42
Sample problems: pages 283-284 problems 49, 50, 55, 56, 57

3 Inequalities

See Sections 2.3 and 2.4. You need to know:

- how to manipulate very simple inequalities, and the basic rules, including:
  - adding and substracting a number on both sides does not change the direction of the inequality
  - multiplying/dividing by a positive number on both sides does not change the direction of the inequality
  - multiplying/dividing by a negative number on both sides *changes* the direction of the inequality
- when applying the same rule on both sides of an inequality, whether the direction of the inequality changes or not depends on the slope of the graph of the rule. For example:
  - applying a rule with positive slope (increasing function) does not change the direction of the inequality
  - applying a rule with negative slope (decreasing function) changes the direction of the inequality
  - if the rule sometimes increases, sometimes decrases, you cannot know.
- how to graph an inequality to solve it (i.e. graph the functions on both sides of the inequality, and find the interval of interest by solving an equality)

Sample problems: page 120 problems 31-60.

4 Power laws

- You need to know the basic grahs of all power laws ($f(x) = x^a$) for every possible value of a. This includes
  - Knowing the domain of definition
  - Knowing the behavior near infinity (when $x \to -\infty$ and $x \to +\infty$)
  - Knowing whether there are asymptotes, and what is the behavior near the asymptote
  - Knowing how to construct functions from that $x^a + b$, bx^a, $(x - b)^a$
- You need to know the formulas for the manipulations of power laws (see Formulas handout)
- You need to know how to find the inverses of the power functions, and solve basic equations involving these powers.

Sample problems:

- Given the function $f(x) = (x+1)^{1/3}$. What is the domain of definition? What is the inverse of this function? Graph $f(x)$ and $f^{-1}(x)$ on the same graph.
- Page A-25, problems 45-50

5 Exponentials and logarithms

Exponentials: See Sections 5.1 and 5.2. You need to know

- The basic expression for an exponential function $f(x) = a^x$ and NOT mix it up with a power function.
- The graphs of basic exponentials including the natural exponential, both in the form $f(x) = a^x$, $f(x) = a^{-x}$ and $f(x) = \left(\frac{1}{a}\right)^x$. This includes the horizontal asymptote (at $y = 0$) and the $y-$intercept $f(0) = 1$.
- How these graphs change when manipulating the exponential a^{x+b}, $b \times a^x$, $a^x + b$, etc...
- How to manipulate these exponentials (see Formulas sheet) and simplify expressions containing exponentials.
- What the natural exponential is.

Sample Problems: page 334, problem 17-32

Logarithm: See Sections 5.3 and 5.4. You need to

- That the logarithm in base a is the inverse of the exponential in base a
- The consequences of that fact in terms of
  - Special values of the log: $\log_a(1) = 0$ and $\log_a(a) = 1$ for any a
  - The graphs of all logarithmic functions: you should be able to graph any basic log function, including the special points above, and the vertical asymptote at $x = 0$.
  - The graphs of related functions $\log_a(x + b)$, $b \log_a(x)$, etc.
  - The domain of the log functions (logs are not defined for $x \leq 0$ so the domain is $x > 0$)
  - The two fundamental equations: $a^{\log_a(x)} = x$ and $\log_a(a^x) = x$.
- How to manipulate logs, and in particular the log properties (see Formulas Handout) and how to use them.
- How to evaluate the logs of simple numbers.
- What the natural logarithm is and how it relates to the natural exponential.

Sample problems:

- page 357, problems 17, 18, 21, 22, 25, 26.
- page 366, problems 11-24

Changes of base: Section 5.4. You need to know the formulas for changing from one base to another, and how to use them. See Formulas Handout.

Sample problems: page 367, 51-61

Solving equations with logs and exponentials: Sections 5.4 and 5.5. You need to be comfortable enough manipulating logs and exponentials, and know their graphs, to be able to solve a whole range of equations involving them. Equations of the kind:

- Basic ones like
  - $a^x = b$
  - $\log_a(x) = b$
- Equations of the kind $a^{a^x} = ...$ and $\log_a(\log_a(x)) = ...$
- Equations of the kind $a^{f(x)} = b^{g(x)}$ and $\log_a(f(x)) = \log_b(f(x))$
- Equations of the kind $9^x + 5 \times 3^x - 4 = 0$, etc (noting that $9^x = (3^x)^2$ and using a change of variable)

Sample problems: page 380 problems 1-40

6 Trigonometric functions

Basic angles, basic trigonometric functions, and the unit circle: See Chapter 7.

- You need to know the basic angles, and in particular:
  - how to go back and forth between radian and degrees for an angle
  - what the unit circle is, and how to visualize angles on the unit circle
  - that angles are defined within $2k\pi$.
- You need to know the two basic functions $\sin(a)$, $\cos(a)$ and derived functions such as $\tan(a)$, $\sec(a)$, $\cot(a)$ and $\csc(a)$ and more specifically:
  - how to visualize $\cos(a)$ and $\sin(a)$ on the unit circle
  - *why* it works
  - that it implies $\cos^2(a) + \sin^2(a) = 1$, *why*, and how to use this formula.
  - the values of sin and cos for the angles $0, \pi/6, \pi/4, \pi/3, \pi/2$ and all other angles which can by constructed from these ones by symmetry.
  - How to deduce $\tan(a)$, $\sec(a)$, $\cot(a)$ and $\csc(a)$ from these values
- You need to be able to simplify simple trigonometric expressions which use these functions.
- You need to be able to graphs and annotate the graphs of
  - $\cos(x)$
  - $\sin(x)$
  - $\tan(x)$
- You need to be able to deduce from these graphs the graphs of the inverse functions $\arccos(x)$, $\arcsin(x)$ and $\arctan(x)$
- You need to be able to sketch the graphs of functions derived from cos and sin such as $A\cos(bx + c)$ and $A\sin(bx + c)$.

- You need to know the definition of a periodic function, and how to find the period of $\cos(bx + c)$ and $\sin(bx + c)$

Sample Problems:

- Homework 8 (trig question), page 515 problems 1-10
- page 516 problems 41-52
- page 532 problems 1-6 (also, what could this function be?), and page 546 problems 16-31, and page 568 problems 1-4

Trignometric formulas. See Chapter 8. You need to know, and know how to use the basic trigonometric formulas (see Formulas handout)

- The Pythagorean formula (see above)
- The double angle formulas for sin and cos
- How to use the addition formulas to expand $\cos(a + b)$ and similar expressions, and to find products of expressions such as $\cos(a)\cos(b)$, etc..

Sample Problems:

- page 584-586 problems 1-12, 33-36, 77-80
- page 596 problems 35-44
- page 603 problems 1-22

Trigonometric equations. Section 8.4. Read and understand the examples in this section.

Sample problems: page 616 problems 5-20