2.2.8 The quadratic formula and quadratic equations

Much of what we have done so far can be generalized to any quadratic function, and this generalization leads to the well-known "quadratic formula" for factoring and finding roots of quadratics.

Step-by-step: From $ax^2 + bx + c$

- Factor a:
- Complete the square in bracket
- Evaluate the discriminant $D = b^2 - 4ac$
  - If $D < 0$ then
  - If $D = 0$

– If $D > 0$

This, for example, gives us a very fast and powerful method for finding solutions of quadratic equations (i.e. equations of the kind $ax^2 + bx + c = 0$:

- Calculate the discriminant $D = b^2 - 4ac$
- If $D < 0$ there are no solutions to the equation, and the quadratic cannot be factored.
- If $D = 0$ there is one solution to the equation, $x = -\frac{b}{2a}$ and the quadratic can be factored as $a\left(x + \frac{b}{2a}\right)^2$
- If $D > 0$ there are two solutions $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$ and the quadratic can be factored as $a(x-x_1)(x-x_2)$

Examples:

- What are the solutions (if any) to the equation $f(x) = 2x^2 - 3x + 1 = 0$? What is the factored form of f?

- What are the solutions (if any) to the equation $f(x) = x^2 + x - 6 = 0$? What is the factored form of f?

- What are the solutions (if any) to the equation $f(x) = -2x^2 - 8x - 8 = 0$? What is the factored form of f?

- What are the solutions (if any) to the equation $f(x) = -x^2 + x - 6 = 0$? What is the factored form of f?

NOTE: In a few particular cases, this method can also help solve higher-order equations that can be reduced to a quadratic, as in these examples:

- What are the solutions (if any) to the equation $f(x) = x^6 - 3x^3 - 9 = 0$?

- What are the solutions (if any) to the equation $f(x) = x^4 - 2x^2 - 3 = 0$?

2.2.9 A fun application for quadratics

Quadratics were first studied seriously when it was realized that they universally describe the trajectory of thrown objects (c.f. Isaac Newton's work). Let's consider the following scenario...

If the white bird is thrown from 1m off the ground, at a velocity of 1m/s, and at a 45 degree angle from the horizontal, how far ahead will it land?

To answer this question, it may help to know that the trajectory of an object thrown at a velocity v_0 (in m/s), and angle α from the horizontal, and from a height h_0, is given by