Chapter 2

Polynomial functions

In this Chapter we will study a large class of functions called "polynomial functions", which all take the form:

The simplest example of these polynomial functions are the linear functions.

2.1 Linear functions

Textbook Section 4.1

2.1.1 Definition and basic properties of linear functions

Definition:

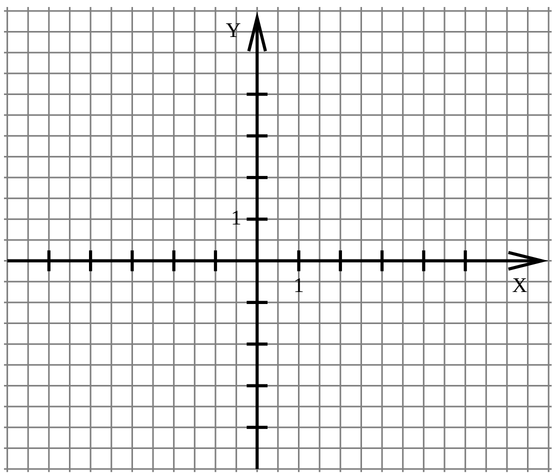
Graph of a linear function: The graph of $y = f(x) = ax + b$ is a straight line with *slope* a and $y-$*intercept* b:

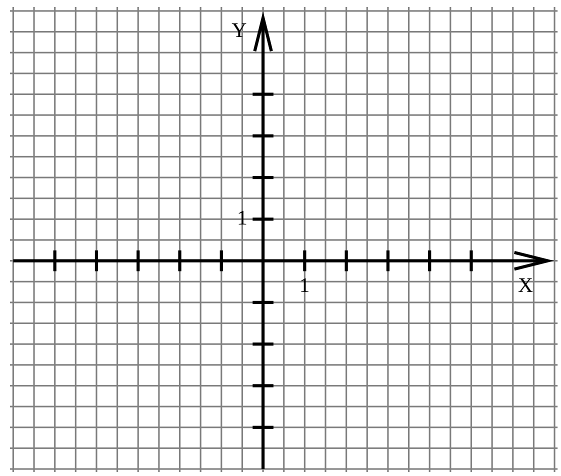
- b is the $y-$intercept

- a is the slope

Note that we can also define the $x-$intercept:

Examples:





Basic properties of a linear function:

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2.1.2 REVIEW: Properties of lines (a little bit of geometry)

Textbook Section 1.6

Studying linear functions is easier if we remember a little bit about properties of lines, since the graph of a linear function *is* a line.

Geometrically speaking, a line is *uniquely* defined either by

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CONSEQUENCE:

Note that if you know the coordinates of the two points, you can calculate the slope of the line going through the points:

Also, remember that

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Once the slope of a line is known, there are two ways of writing the line equation:

- The slope-intercept formula (if you know the $y-$intercept):
- The point-slope formula (if you know a point on the line):

Examples:

- Finding a line going between two points:

- Finding a line going through one point, with a "given" slope:

2.1.3 Practical uses of linear functions

Textbook Section 4.1

Linear functions are useful in many different ways, and are the most commonly-used functions for modeling of any scientific field. They may arise as *exact* representations of a linear relationship between two variables, or as *models* for the *apparently* linear relationship between two observed variables. We will now see an example of each...

Exact representations of linear relationships

We have already seen an exact linear function in the Toy Story III example. Linear functions are also very common when we try to model the cost of production of some object (in ecomomics), because typically the total costs are split between a "basic setup" cost, and a "cost per unit" produced. It can also easily be used to model the income per sale of an object, and ultimately, help select pricing.

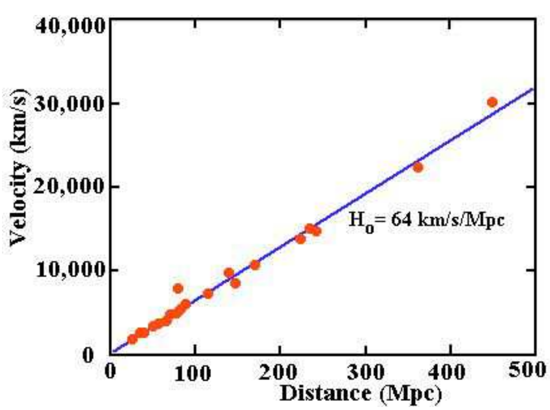
Example: Donelly's (on Mission) produce high-quality chocolates truffles. The cost of production of the truffles is split between the initial purchase of the instruments needed (refrigerators, high-precision thermometers, containers, chocolate-melting and mixing devices, etc...) which amounts to \$10,000 and the cost of the basic ingredients for each truffle (estimated at 50c/truffle). What is the total production cost for x truffles, in dollars?

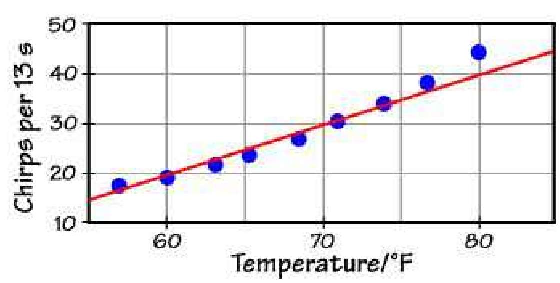
Suppose the sale price of each truffle is p. What is the total income as a function of the number of truffles sold?

What guides the pricing selection for the truffles?

Modeling apparently linearly-related variables

In Lecture 1, we saw that there are quite a few examples in real natural systems where two variables are apparently linearly related.





In these particular cases, trend in the real data appears to be linear, although, perhaps through errors in the measurements or other "contaminating" effects, the data points don't lie *exactly* along a straight line.

Scientists are typically interested in finding the best "linear fit" to the data. This is called doing a *linear regression* and is studied in detail in Statistics (see AMS 5 and AMS 7). Here, we will simply do it by taking 2 "well-chosen" data points.

- The Hubble plot:

- The cricket plot:

Once we have the data fitted to a linear function, we can predict what the values of the function may be for values of the independent variables that have not been measured. This is called *linear interpolation* (if the new point lies within the range of the existing data set), or *linear extrapolation* (if the new point lies outside the range of the existing data set).

Examples:

- What would be the receeding velocity of a galaxy 300Mpc away?

- What would be the chirp-rate of a cricket at 50°?