3.2 Inequalities

Textbook section 2.3

Now that we know a fairly extensive range of basic functions, we have the tools to study inequalities in more detail.

3.2.1 What does "solving" inequalities mean?

DEFINITION:

EXAMPLES:

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There are two basic methods for solving inequalities. One relies on knowing the graphs of basic functions and solving equations, the second relies on knowing how to manipulate inequalities to isolate the variable.

3.2.2 "Graphical" method for solving inequalities

A very powerful method for solving inequalities consists in looking at the graphs of the functions $f(x)$ and $g(x)$:

As a result, in order to solve the inequality:

EXAMPLE: Solve the inequality $x^2 + 2x + 1 \geq 1$

EXAMPLE: Solve the inequality $|x + 1| \geq -2x$

EXAMPLE: Solve the inequality $\frac{3x-2}{x+1} \leq 1$

3.2.3 "Direct" method for solving inequalities

The direct method for solving inequalities relies on isolating the variable x, when possible, on one side of the inequality. Note that this is not always possible, but when it is possible, it can be quite useful. It is also important to know the rules of manipulations of inequalities.

THE RULES OF MANIPULATIONS OF INEQUALITIES. The standard rules are the following:

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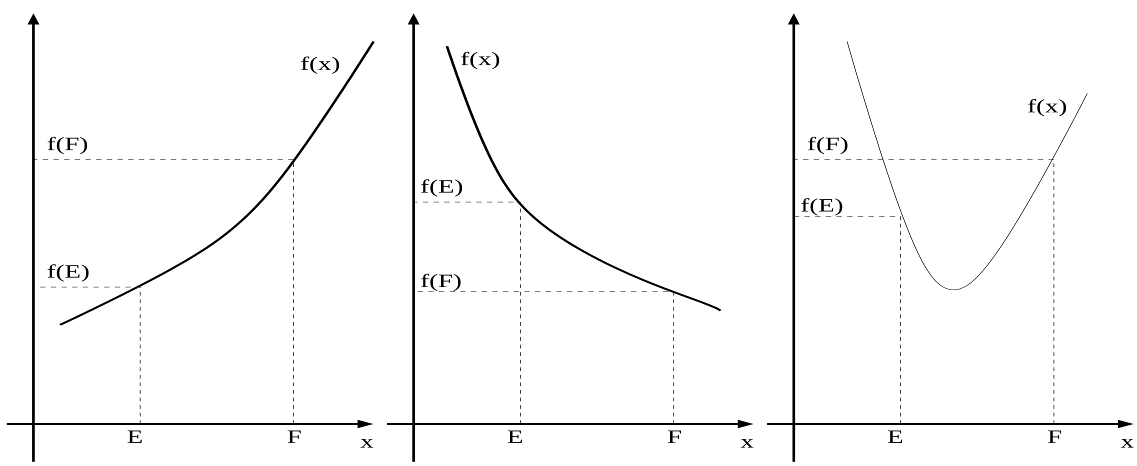
EXAMPLES:

BUT WHERE DO THESE COME FROM, AND HOW TO DEAL WITH MORE COMPLICATED PROBLEMS?

Imagine you have an inequality $E \leq F$ where E and F can be any expressions (numbers, variables, functions, etc..). Then, if you want to transform the inequality with a transformation rule f, note that

- $E \leq F \Rightarrow f(E) \leq f(F)$ if f is an increasing function in $[E, F]$
- $E \leq F \Rightarrow f(E) \geq f(F)$ if f is a decreasing function in $[E, F]$
- We don't know what happens if f has a slope that changes in $[E, F]$.



EXAMPLES:

- You want to add a constant C to both sides of an inequality.

- You want to multiply by a constant C both sides of an inequality.

MORE EXAMPLES: Suppose you have the inequality

$$x \leq y$$

and both x and y are greater than 0. Using the relevant graph, assess whether the following statements are true or false?

- $x^2 \leq y^2$
- $\frac{1}{x} \leq \frac{1}{y}$
- $\sqrt{x} \geq \sqrt{y}$
- $3x + 2 \leq 3y + 2$
- $(x + 1)^2 - 5 \leq (y + 1)^2 - 5$