Handout: Summary of AMS 3 formulas

All formula with an asterisk must be known by heart. All others are optional and will be given to you in exams if required.

1 Lines, circles and points

The equation of a line with slope s and y-intercept b is

$$y = f(x) = sx + b \qquad (*) \tag{1}$$

If the line goes through 2 points A and B with coordinates (x_A, y_A) and (x_B, y_B) then

$$s = \frac{y_B - y_A}{x_B - x_A} \qquad (*) \tag{2}$$

The distance between two points $A(x_A, y_A)$ and $B(x_B, y_B)$ is

$$d = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \qquad (*) \tag{3}$$

The equation of a circle of radius R centered on $A(x_A, y_A)$ is

$$(x - x_A)^2 + (y - y_A)^2 = R^2 \qquad (*) \tag{4}$$

2 Quadratic equations

$$y = f(x) = ax^2 + bx + c \qquad (*) \tag{5}$$

The graph of $y = f(x)$ is a parabola. It has a minimum (i.e. parabola opens upwards) if $a > 0$. It has a maximum (i.e. parabola opens downwards) if $a < 0$. The minimum/maximum is at the location x_m with

$$x_m = -\frac{b}{2a} \qquad (*) \tag{6}$$

It has roots (i.e it intersects the x-axis) when $y = f(x) = 0$. The solutions to this equation depends on the value of D:

$$D = b^2 - 4ac \qquad (*) \tag{7}$$

- if $D < 0$ there are no solutions. The parabola does not intercept the x-axis. The function $f(x) = ax^2 + bx + c$ cannot be factored.
- if $D = 0$ there is one solution. The parabola just touches the x-axis at the point

  $$x_1 = x_m = -\frac{b}{2a} \qquad (*) \tag{8}$$

  The function $f(x) = ax^2 + bx + c$ is factored as

  $$f(x) = a(x - x_1)^2 \qquad (*) \tag{9}$$

- if $D > 0$ there are two solutions. The parabola intercepts the x-axis in the two points

$$x_1 = \frac{-b - \sqrt{D}}{2a}\ ,\ x_2 = \frac{-b + \sqrt{D}}{2a} \qquad (*) \tag{10}$$

The function $f(x) = ax^2 + bx + c$ is factored as

$$f(x) = a(x - x_1)(x - x_2) \qquad (*) \tag{11}$$

3 Polynomial functions

$$y = f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n \qquad (*) \tag{12}$$

a_n is the leading coefficient, n is the order of the polynomial.

The factored form of f is

$$f(x) = a_n(x - x_1)(x - x_2)(x - x_3)\ldots(x - x_m)q(x) \qquad (*) \tag{13}$$

where x_i are all possible solutions to $f(x) = 0$ and $q(x)$ is a polynomial of order $n - m$, leading coefficient 1, with no roots ($q(x) \neq 0$).

4 Rational functions

$$y = f(x) = \frac{p(x)}{q(x)} \qquad (*) \tag{14}$$

where $p(x)$ and $q(x)$ are polynomial functions.

The roots of $p(x)$ are the roots of $f(x)$. The roots of $q(x)$ are the asymptotes of $f(x)$.

5 Power functions

$$y = f(x) = x^a \qquad (*) \tag{15}$$

Properties:

$$x^{a+b} = x^a x^b \qquad (*) \tag{16}$$

$$x^{-a} = \frac{1}{x^a} \qquad (*) \tag{17}$$

$$x^{a-b} = \frac{x^a}{x^b} \qquad (*) \tag{18}$$

$$x^{ab} = (x^a)^b = (x^b)^a \qquad (*) \tag{19}$$

6 Exponential functions

Exponential in base a:

$$y = f(x) = a^x \text{ with } a > 0 \qquad (*) \tag{20}$$

Natural exponential (exponential in base e with $e = 2.71828\ldots$):

$$y = f(x) = e^x = \exp(x) \qquad (*) \tag{21}$$

Properties of all exponential functions:

$$a^{x+z} = a^x a^z \qquad (*) \tag{22}$$

$$a^{-x} = \frac{1}{a^x} \qquad (*) \tag{23}$$

$$a^{x-z} = \frac{a^x}{a^z} \qquad (*) \tag{24}$$

$$a^{xz} = (a^x)^z = (a^z)^x \qquad (*) \tag{25}$$

7 Logarithmic functions

Logarithm in base a is the inverse of the exponential in base a:

$$y = \log_a(x) \text{ is equivalent to } x = a^y \qquad (*) \tag{26}$$

Natural logarithm (logarithm in base e) is the inverse of the natural logarithm:

$$y = \log_e(x) = \ln(x) \text{ is equivalent to } x = e^y \qquad (*) \tag{27}$$

Inverse relations:

$$\log_a(a^x) = x \qquad (*) \tag{28}$$

$$a^{\log_a(x)} = x \qquad (*) \tag{29}$$

$$\ln(e^x) = x \qquad (*) \tag{30}$$

$$e^{\ln(x)} = x \qquad (*) \tag{31}$$

Properties of all logarithmic functions:

$$\log_a(xy) = \log_a(x) + \log_a(y) \qquad (*) \tag{32}$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a(x) \qquad (*) \tag{33}$$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y) \qquad (*) \tag{34}$$

$$\log_a(x^c) = c\log_a(x) \text{ where c is a positive constant} \qquad (*) \tag{35}$$

Relations for changing bases:

- From an exponential function in base a to the natural exponential:

$$a^x = e^{x\ln a} \tag{36}$$

- From a logarithmic function in base a to the natural logarithm:

$$\log_a(x) = \frac{\ln x}{\ln a} \tag{37}$$

8 Trigonometric functions

The basic trigonometric functions are:

$$y = f(x) = \sin(x) \qquad (*) \tag{38}$$

$$y = f(x) = \cos(x) \qquad (*) \tag{39}$$

$$y = f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)} \qquad (*) \tag{40}$$

$$y = f(x) = \cot(x) = \frac{\cos(x)}{\sin(x)} \qquad (*) \tag{41}$$

$$y = f(x) = \sec(x) = \frac{1}{\cos(x)} \qquad (*) \tag{42}$$

$$y = f(x) = \csc(x) = \frac{1}{\sin(x)} \qquad (*) \tag{43}$$

$$\tag{44}$$

Table of values you have you know (*):

Angle (degree)	Angle (radian)	sin	cos	tan
0	0	0	1	0
30	$\pi/6$	0.5	$\sqrt{3}/2$	$1/\sqrt{3}$
45	$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1
60	$\pi/3$	$\sqrt{3}/2$	0.5	$\sqrt{3}$
90	$\pi/2$	1	0	not defined

Properties:

$$\cos^2 x + \sin^2 x = 1 \qquad (*) \tag{45}$$

$$\sin(2x) = 2 \sin x \cos x \qquad (*) \tag{46}$$

$$\cos(2x) = \cos^2 x - \sin^2 x \qquad (*) \tag{47}$$

Other addition/multiplication formula

$$\cos(a + b) = \cos a \cos b - \sin a \sin b \tag{48}$$

$$\cos(a - b) = \cos a \cos b + \sin a \sin b \tag{49}$$

$$\sin(a + b) = \sin a \cos b + \cos a \sin b \tag{50}$$

$$\sin(a - b) = \sin a \cos b - \cos a \sin b \tag{51}$$