AMS 3: Midterm 2011

Name: ________________________________

Calculators are not allowed.

Read all the questions before you start working on any of them. Start with the ones you are most comfortable with, and continue with the other ones later. Always double-check your answers. If you need scrap paper, please ask the instructor/proctor.

Relax, and do your best!

PROBLEM 1: SHORT QUESTIONS. In the following questions, you are merely asked to provide the answer. No justification is needed. You should not be spending more than 1 minute per question.

1. What is the equation of the line passing through the points $(1, 1)$ and $(-2, 2)$? Write it in the form $y = ax + b$.

2. Find the linear function $f(x)$ such that $f(0) = 0$ and $f(3) = 2$.

Given the functions $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{1-x}$

3. Write down, and then simplify the expression $f(x)-f(x-1)$. ______________________________________

4. What is the domain of $f(x)$? ______________________________________

5. What is the domain of $g(x)$? ______________________________________

6. What is $f \circ g(x)$? ______________________________________

7. What is $g \circ f(x)$? ______________________________________

8. What is the inverse of $g(x)$? ______________________________________

9, 10. Sketch the function $k(x) = -(x+1)^3 + 1$ **and** its inverse on the same graph.

11. If $f(x) = \frac{1}{x}$, what is $f[f^{-1}(2x^2)]$? ____

12. Complete the square for the expression $-x^2+2x+3$: ____

Given the parabola $y = -(x+1)^2 + 4$:

13. What are the coordinates of the vertex? ____

14. Does it open up or down? ____

15. What is the $y-$intercept? ____

16. What are the $x-$intercepts? ____

17. Based on this information, sketch the parabola $y = -(x+1)^2 + 4$, making sure to annotate your graph correctly.

18. How many times does this parabola intercept the $x-$axis?

$f(x) = x^2 + \sqrt{3}x + 3$? ____

19. Factor the quadratic $-x^2 + 2x + 5$

20. Factor the following expression by grouping, and make sure your result is *fully* factored:
$2x^4 - 4x^3 - x^2 + 2x$.

PROBLEM 2: FUNCTIONS AND INVERSES: Consider the function $f(x) = x^2 - 2x + 1$.

(a) What is the name of this kind of function? ____

(b) What are the solutions to $f(x) = 0$? ____

(c) Factor $f(x)$, then draw a signs table for $f(x)$

(d) Using this information, sketch $f(x)$. Note: your $y-$intercept must be correct and annotated.

(e) Explain why, when finding the inverse, we should limit our study to the interval $x \geq 1$.

(f) Solve the equation $x^2 - 2x + 1 = y$ for x, and, by applying the confition $x \geq 1$, decide which of the two possible solutions to keep.

(g) Deduce what $f^{-1}(x)$ is: ____

(h) What is the domain of $f^{-1}(x)$? ____

(i) Sketch the functions $f(x)$ (for $x \geq 1$) and $f^{-1}(x)$ (for x in the domain of f^{-1}) on the same graph. Clearly mark which is which.

PROBLEM 3. HIGHER ORDER POLYNOMIAL Consider the higher order polynomial function $-x^3+3x^2-2x$.

(a) What is the behavior near $+\infty$ and $-\infty$?
When x tends to $-\infty$, $f(x)$ goes to ____________________
When x tends to $+\infty$, $f(x)$ goes to ____________________

(b) Factor the function: ______________________________________

(c) Determine the $x-$ and $y-$ intercepts

$x-$intercept(s): ___________________________ $y-$intercept: ___________________________

(d) Draw a signs table

(e) Sketch the function

Problem 4. Applied Problem.

This problem guides you through a mathematical proof that the largest possible rectangular area of perimeter 1 (one) is achieved when that rectangular area is actually a square.

Question 1: Considering a rectangle of length x and width y. How does y relate to x given the constraints of the problem?

Question 2: Show that the area of the rectangle as a function of x is $A(x) = \frac{1}{2}x - x^2$

Question 3: What is the name of this type of function? _______________

Question 4: Using the standard methods we have learned, sketch this function (Hint: find the $x-$ and $y-$intercepts, etc..)

Question 5: Using the standard methods we have learned, find the $x-$position of the maximum of this function.

Question 6: Calculate the y that corresponds to this optimal x. Why does this mean the rectangle is a square?