HW9 Note: Trig functions on last two pages

Section 5.4: 42, 44, 46, 48, 52, 54, 56, 58, 60, 70, 76, 78, 82

42. Graph equations and determine x-intercepts

a) $y = 3^{x-1} - 2$

$0 = 3^{x-1} - 2$

$2 = 3^{x-1}$

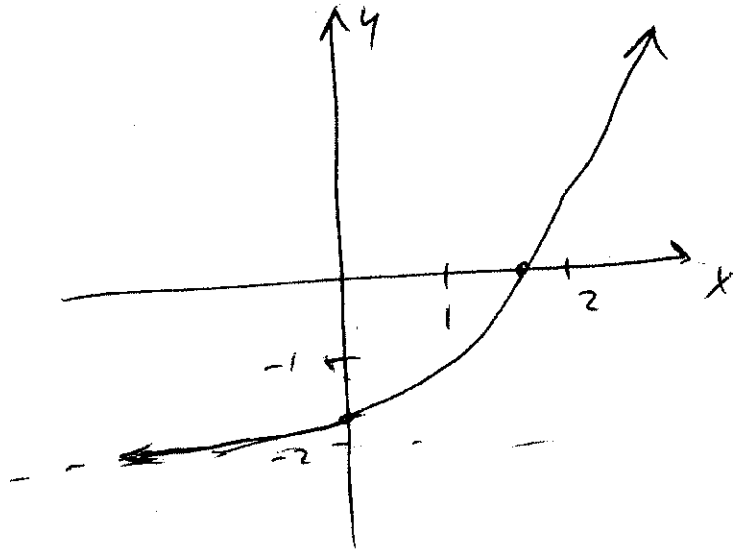
$\ln 2 = (x-1)\ln 3$

$\frac{\ln 2}{\ln 3} + 1 = x$

$x \approx 1.63$

$y = 3^{-1} - 2$

$y\text{-int} = +\frac{1}{3} - 2 = -1\frac{2}{3}$



[20]

b) $y = 3^{1-x} - 4$

$0 = 3^{1-x} - 4$

$4 = 3^{1-x}$

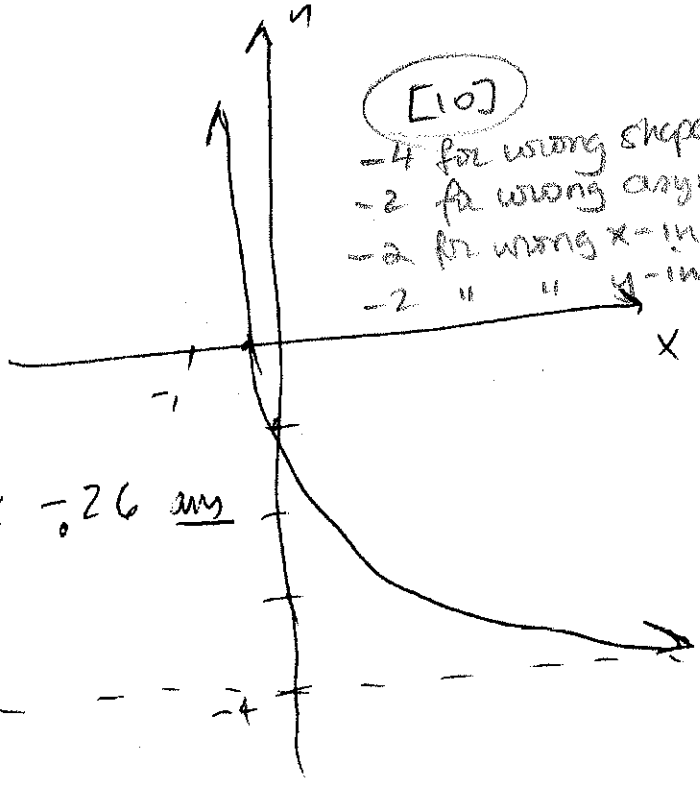
$\ln 4 = (1-x)\ln 3$

$\frac{\ln 4}{\ln 3} - 1 = -x$

[10] $\boxed{x = 1 - \frac{\ln 4}{\ln 3}} \approx -.26$ ans

$y = 3^{1-0} - 4$

-3/ algebra error $y = -1$



[10]
-4 for wrong shape
-2 for wrong asymptote
-2 for wrong x-int
-2 " " y-int

44. $3e^{1+t}=2$

$\ln 3+\ln e^{1+t}=\ln 2$

$\ln 3+(1+t)=\ln 2$

$1+t=\ln 2-\ln 3$

$t=\ln 2-\ln 3-1$

46. $5^{3x-1}=27$

$(3x-1)\ln 5=\ln 27$

$(3x-1)=\frac{\ln 27}{\ln 5}$

$3x=\frac{\ln 27}{\ln 5}+1$

$x=\frac{1}{3}\left(\frac{\ln 27}{\ln 5}+1\right)$

48 $10^{2x+3}=280$

$(2x+3)\ln 10=\ln 280$

$2x+3=\frac{\ln 280}{\ln 10}$

$2x=\frac{\ln 280}{\ln 10}-3$

$x=\frac{1}{2}\left(\frac{\ln 280}{\ln 10}-3\right)$

52. $\log_5 10 = \frac{\log_{10} 10}{\log_{10} 5} = \frac{1}{\log 5}$ ans

54. $\ln 10 = \frac{\log_{10} 10}{\log_{10} e} = \frac{1}{\log_{10} e}$ ans

56. $\log_2 b = \frac{\log_{10} b}{\log_{10} 2}$ ans

[10] 56. $\log_2 10 = \frac{\ln 10}{\ln 2}$ ans

60. $\log_b 2$ where $b = e^2 = \frac{\ln 2}{\ln e^2} = \frac{\ln 2}{2}$ ans

[20] 70. $(1, 2)$ & $(4, 8)$ on $y = a^{bx}$

$y = ae^{bx}$

$2 = ae^{b}$ $\quad 8 = ae^{4b}$

Dividing $\frac{8}{2} = \frac{ae^{4b}}{ae^{b}}$

$4 = e^{3b}$

$\ln 4 = 3b$

$\boxed{b = \frac{\ln 4}{3}}$ ans

$2 = ae^{\ln 4/3}$

$\boxed{a = \frac{2}{4^{1/3}}}$ ans

-5 algebra error

76. Is there a constant k such that

$$e^x = 2^{kx} \quad \text{for all } x?$$

Yes ans since $e^x = (2^k)^x$

$$e = 2^k$$

$$1 = k \ln 2 \Rightarrow k = \frac{1}{\ln 2}$$

78. Simplify $(\log_2 3)(\log_3 4)(\log_4 5)$

$$\log_2 3 \left(\frac{\log_2 4}{\log_2 3}\right)\left(\frac{\log_2 5}{\log_2 4}\right) = \log_2 5 \text{ ans}$$

82. Simplify a^x when $x = \dfrac{\log_b(\log_b a)}{\log_b a}$

$$a^{\log_b(\log_b a)/\log_b a} =$$

$$\left(b^{\log_b a}\right)^{\log_b(\log_b a)/\log_b a} =$$

$$b^{\log_b(\log_b a)} = \log_b a \text{ ans}$$

Section 5.5 2, 4, 6, 8, 10, 32, 40, 80, 82, 84

2: $7^{-4x} = 2^{1+3x}$

$$-4x \ln 7 = (1+3x)\ln 2$$

$$(1+3x)\ln 2 = -4x \ln 7$$

$$3x \ln 2 = -4x \ln 7 - \ln 2$$

$$3x \ln 2 + 4x \ln 7 = -\ln 2$$

$$x(3\ln 2 + 4\ln 7) = -\ln 2$$

$$x = \frac{-\ln 2}{3\ln 2 + 4\ln 7} \text{ ans}$$

$$x \approx -0.0703 \text{ ans}$$

4. $\log_3(\log_3(2x)) = -2$

$\log_3(2x) = 3^{-2}$

$\log_3(2x) = \frac{1}{9}$

$2x = 3^{1/9}$

$x = \frac{3^{1/9}}{2} \approx 0.565$ ans

[10] 6. $\log_2(2x^2 - 4) = 5$

$2x^2 - 4 = 2^5$

$2x^2 = 36$

$x^2 = 18$

$x = \pm 3\sqrt{2}$ ans or $\pm\sqrt{18}$ (-1 if forgot ±).

$x \approx \pm 4.243$ ans

-3/algebra error

8. $\log_9(x^2 + x) = 0.5$

$x^2 + x = 9^{1/2}$

$x^2 + x = 3$

$x^2 + x - 3 = 0$

$x = \frac{-1 \pm \sqrt{1+12}}{2} = -\frac{1}{2} \pm \frac{1}{2}\sqrt{13} = -2.303$ or 1.303 ans

[20] 10. $3(2^{2x}) - 11(2^x) - 4 = 0$ let $t = 2^x$

$3t^2 - 11t - 4 = 0$

$(3t + 1)(t - 4) = 0$

$3t + 1 = 0 \quad t - 4 = 0$

$t = -1/3 \quad t = 4$

$2^x = -1/3$ not a solution

$2^x = 4$

$2^x = 2^2$

$\boxed{x = 2}$ ans

-4/algebra error

32. $\log_6 x + \log_6 (x+1) = 0$

$\log_6 x(x+1) = 0$

$x(x+1) = 1$

$x^2 + x - 1 = 0$

$x = \frac{-1 \pm \sqrt{1+4}}{2}$

$x = \frac{-1}{2} + \frac{\sqrt{5}}{2}$ ans (negative solution extraneous makes log argument < 0)

40. $\log_2 (2x^2 + 4) = 5$

$2x^2 + 4 = 2^5$

$2x^2 + 4 = 32$

$2x^2 = 28$

$x^2 = 14$

$x^2 = \pm\sqrt{14}$ ans

80. $3 \ln x = \alpha + 3 \ln \beta$

$3 \ln x - 3 \ln \beta = \alpha$

$\ln x - \ln \beta = \alpha/3$

$\ln \frac{x}{\beta} = \alpha/3$

$\frac{x}{\beta} = e^{\alpha/3}$

$x = \beta e^{\alpha/3}$ ans

[10] 82. $\beta = 10 \log_{10}(x/x_0)$

$\frac{\beta}{10} = \log_{10}\left(\frac{x}{x_0}\right)$

$10^{\beta/10} = \frac{x}{x_0}$

$x = x_0 10^{\beta/10}$ ans

84. $T = T_1 + (T_0 - T_1)e^{-kx}$

$T - T_1 = (T_0 - T_1)e^{-kx}$

$e^{-kx} = \frac{T - T_1}{T_0 - T_1}$

$-kx = \ln\left(\frac{T - T_1}{T_0 - T_1}\right)$

$x = -\frac{1}{k}\ln\left(\frac{T - T_1}{T_0 - T_1}\right)$ ans

Applied Problem

Question 1. (1974) $4.088 - .0175(4.088) = 4.0165$. ans $(t=-1$

(1976) $4.088 + .0175(4.088) = 4.1595$ Ans $(t=1$

(1977) $4.1595 + .0175(4.1595) = 4.233$ Ans $(t=2)$

Question 2. $N = N_0 a^{t-1975}$ $N_0 = 4.088$ $t = 1976$ $N = 4.1595$

$4.1595 = 4.088\, a^{\underbrace{1976-1975}_{1}}$

$a = 1.0177$ ans

$N_0 = 4.088$ ans

$N = 4.088(1.0177)^{t-1975}$ ans

Question 3. $N = (4.088)(1.0177)^{2009-1975}$

$N = 4.088 \cdot 1.0177^{34}$

$N = 7.423$ billion ans

Question 4 $N = N_0 e^{0.0175(t-1975)}$ ans

since $a = e^r$ $\ln a = r \Rightarrow \ln(1.0177) = 0.0175$ ans

r

Question 5. How long does the population take to double?

$2N_0 = N_0 e^{0.0175(t-1975)}$ ($t-1975 = T$)

$2 = e^{0.0175T}$

$\ln 2 = 0.0175T$

$T = \frac{\ln 2}{.0175} = 39.6 \approx 40$ years ans

Question 6. Estimate population at year 0

$N = 4.088(1.0177)^{0-1975}$

$N = 4.088(1.0177)^{-1975}$

$N \approx 0$ ans $\approx$ means approximately

Question 7. The world population has a different growth rate at different times. Initially, the growth rate was much smaller apparently.

(8)

Homework 8

See figure below:

This homework is due in class on Monday 11/30/09

1 Course material

Properties of exponential and logarithmic functions:

- Textbook Section 5.4: 42, 44, 46, 48, 52, 54, 56, 58, 60, 70, 76, 78 (Hint: use change of base to the natural logarithm), 82

Solving equations with exponentials and logarithms

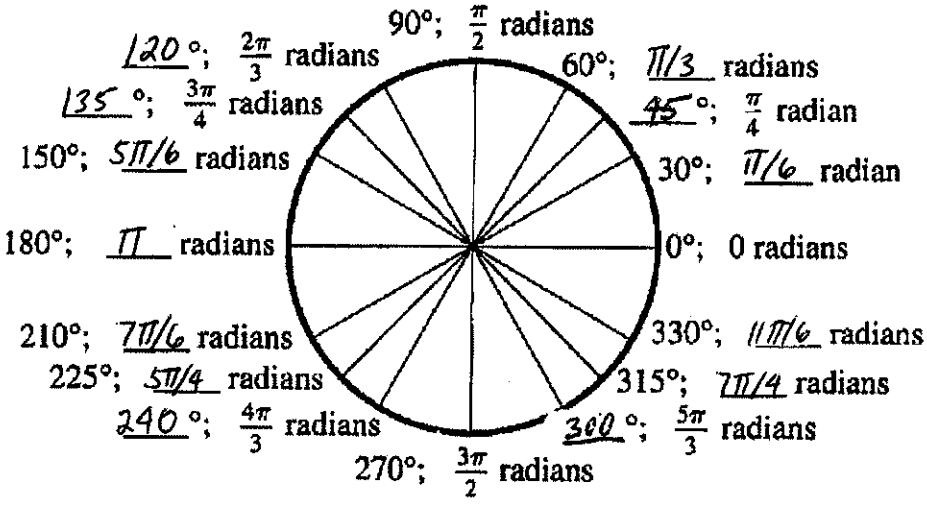
- Textbook Section 5.5: 2, 4, 6, 8, 10, 32, 40, 80, 82, 84

Basic trigonometric functions

In the following figure, complete the missing angles. Give the sine, cosine, tangent, co-tangent, secant and cosecant of each angle.

[10]

(-1/ wrong angle)



Write your answers in a table as in the following example: where ND stands for Not Defined.

Degrees	Radian	cos	sin	tan	cotan	sec	cosec
0	0	1	0	0	ND	1	ND

Table

Degrees	Radians	cos	sin	tan	cotan	sec	cosec
0	0	1	0	0	ND	1	ND
30	$\pi/6$	.866	.5	.577	1.732	1.155	2
45	$\pi/4$	.707	.707	1	1	1.414	1.414
60	$\pi/3$	.5	.866	1.732	.577	2	1.155
90	$\pi/2$	0	1	ND	0	ND	1
120	$2\pi/3$	-.5	.866	-1.732	-.577	-2	1.155
135	$3\pi/4$	-.707	.707	-1	-1	-1.414	1.414
150	$5\pi/6$	-.866	.5	-.577	-1.732	-1.155	2
180	π	-1	0	0	ND	-1	ND
210	$7\pi/6$	-.866	-.5	.577	1.732	-1.155	-2
225	$5\pi/4$	-.707	-.707	1	1	-1.414	-1.414
240	$4\pi/3$	-.5	-.866	1.732	.577	-2	-1.155
270	$3\pi/2$	0	-1	ND	0	ND	-1
300	$5\pi/3$	.5	-.866	-1.732	-.577	2	-1.155
315	$7\pi/4$	.707	-.707	-1	-1	1.414	-1.414
330	$11\pi/6$	.866	-.5	-.577	-1.732	1.155	-2

Note $0.866 = \frac{\sqrt{3}}{2}$

$0.707 = \frac{\sqrt{2}}{2}$

$1.732 = \sqrt{3}$

$0.577 = \frac{1}{\sqrt{3}}$