HOMEWORK 7

Section 2.4: 16 (direct), 22 (direct), 12 (graphical), 32 (graphical), 39 (graphical), 44 (graphical), 54 (graphical), 52 (graphical), 58 (graphical

10

[10] 16. $24 - x^2 \geq 0$

Factor

$(\sqrt{24} - x)(\sqrt{24} + x) \geq 0$

$(2\sqrt{6} - x)(2\sqrt{6} + x) \geq 0$

Or say
$24 \geq x^2$
$\sqrt{24} \geq |x|$ $\sqrt{24} = 2\sqrt{6}$.

$\Rightarrow$ As long as $|x| \leq 2\sqrt{6}$ the inequality is satisfied

So when $x \in [-2\sqrt{6}, 2\sqrt{6}]$ ans

$x \in [-\sqrt{24}, \sqrt{24}]$ OK, $-\sqrt{24} \leq x \leq \sqrt{24}$ ✓

22 $1 + x^2 < 0$

Note $1 + x^2$ always greater than 0

So $1 + x^2 < 0$ is the empty set <u>ans</u>

[10] 12. $2x^2 + 7x + 5 > 0$

Factor $(2x+5)(x+1) > 0$

$2x + 5 = 0 \Longrightarrow x = -5/2$

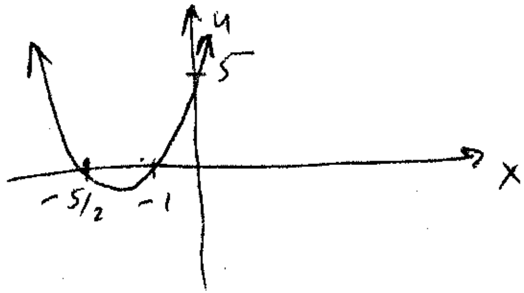
$x + 1 = 0 \Longrightarrow x = -1$ y-intercept $(0+5)(0+1) = 5$

Sign Table

Factor		$-5/2$		-1	
$2x+5$	$-$	0	$+$		$+$
$x+1$	$-$		$-$	0	$+$
$f(x)$	$+$	0	$-$	0	$+$

From sign Table $f(x) > 0$ when $x \in (-\infty, -\frac{5}{2}) \cup (-1, \infty)$ ans.

Graph



32. $(x - \frac{1}{2})(x + \frac{1}{2})(x + \frac{3}{2}) < 0$

x-intercepts = $\{-3/2, -1/2, 1/2\}$

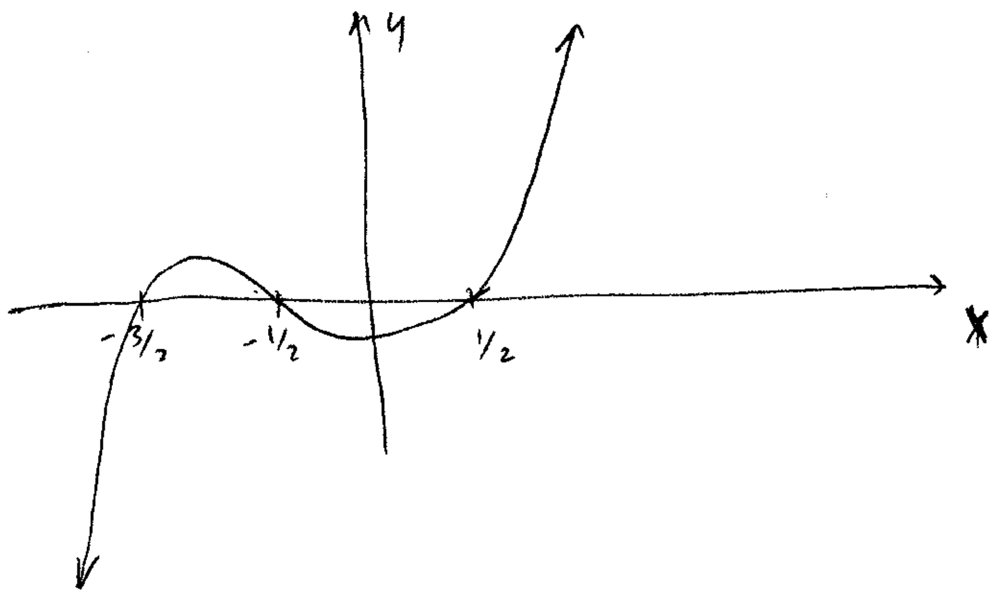
as $x \to +\infty$ $y \to +\infty$, as $x \to -\infty$, $y \to -\infty$

Sign Table

Factor		$-3/2$		$-1/2$		$1/2$	
$x - 1/2$	$-$		$-$		$-$	0	$+$
$x + 1/2$	$-$		$-$	0	$+$		$+$
$x + 3/2$	$-$	0	$+$		$+$		$+$
$f(x)$	$-$	0	$+$	0	$-$	0	$+$

From sign table

$f(x) < 0$ when $x \in (-\infty, -3/2) \cup (-1/2, 1/2)$ ans



39. $9(x-4) - x^2(x-4) < 0$ (12)

Factor

$(9-x^2)(x-4) < 0$ x-intercepts

$(3+x)(3-x)(x-4) < 0$ $x = \{-3, 3, 4\}$

as $x \to +\infty, y \to -\infty$, as $x \to -\infty, y \to +\infty$ $y_{intercept} = (3+0)(3-0)(0-4) = -36$

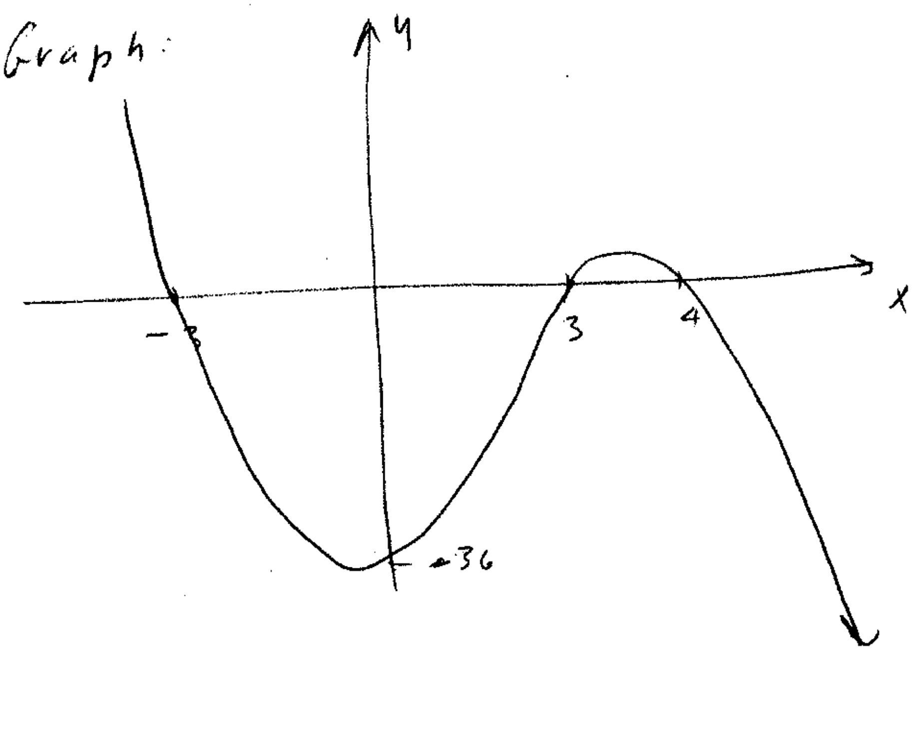
Sign Table

Factor		-3		3		4	
$3+x$	$-$	0	$+$		$+$		$+$
$3-x$	$+$		$+$	0	$-$		$-$
$x-4$	$-$		$-$		$-$	0	$+$
$f(x)$	$+$	0	$-$	0	$+$	0	$-$

From sign Table

$f(x) < 0$ when $x \in (-3, 3) \cup (4, \infty)$ ans

Graph:



44. $\dfrac{x+4}{2x-5} \le 0$

[b]

vertical asymptote $2x-5=0 \Rightarrow x = 5/2$

x-intercept $= -4$ horizontal asymptote $y = 1/2$

y-intercept: $-4/5$

Sign Table

Factor		-4		$5/2$	
$x+4$	$-$	0	$+$		$+$
$2x-5$	$-$		$-$	∞	$+$
$f(x)$	$+$	0	$-$	∞	$+$

From sign table

$\boxed{f(x) \le 0 \text{ when } x \in [-4, 5/2)}$ ans

54. $\dfrac{2}{x} < \dfrac{x}{2}$ Put all on one side

$\dfrac{2}{x} - \dfrac{x}{2} < 0$

$\dfrac{4}{2x} - \dfrac{x^2}{2x} < 0$

$\dfrac{4-x^2}{2x} < 0$

$\dfrac{(2-x)(2+x)}{2x} < 0$

x-intercepts $= \{2, -2\}$

vertical asymptote $x = 0$

as $x \to +\infty$ $y \to -\infty$

as $x \to -\infty$ $y \to +\infty$

Sign Table

Factor		-2		0		2	
$2+x$	$-$	0	$+$		$+$		$+$
$2-x$	$+$		$+$		$+$	0	$-$
$2x$	$-$		$-$	∞	$+$		$+$
$f(x)$	$+$	0	$-$	∞	$+$	0	$-$

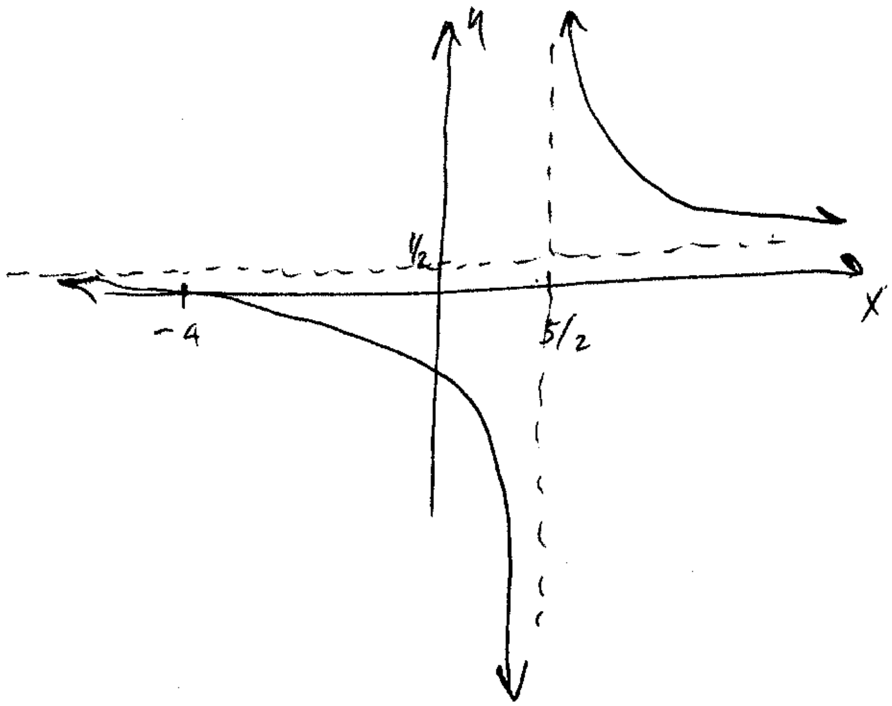
From sign table

inequality holds when $x \in (-2,0) \cup (2,\infty)$ ans

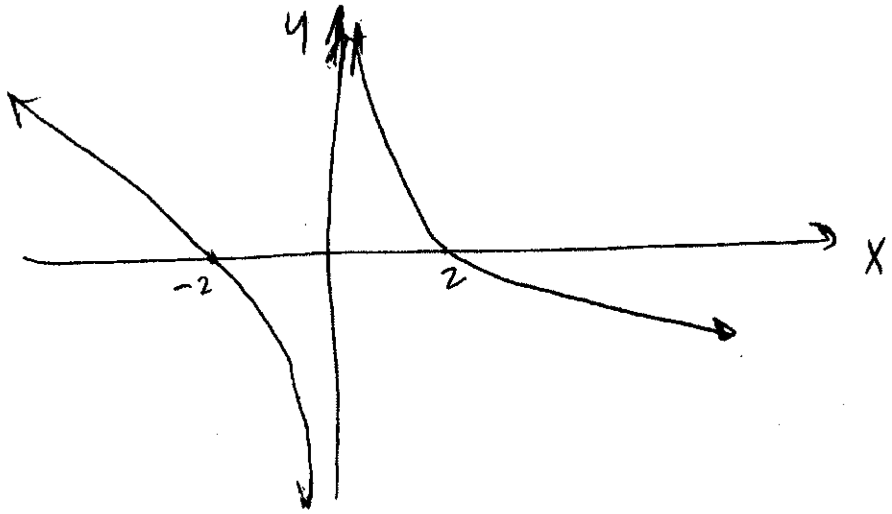
See next page for graphs of 44 & 54

13 B

44. Graph $\frac{x+4}{2x-5} \le 0$



54 Graph $\frac{(2-x)(2+x)}{2x} < 0$



52. $\frac{2x}{x-2} < 3$

Solve to get everything on one side

$$\frac{2x}{x-2} - 3 < 0$$

$$\frac{2x}{x-2} - \frac{3(x-2)}{x-2} < 0$$

$$\frac{2x - 3x + 6}{x-2} < 0$$

$$\frac{6 - x}{x-2} < 0$$

y-intercept : $\frac{6-0}{0-2} = -3$
x-intercept $= 6$
vertical asymptote $= 2$
Horizontal asymtote $y = -1$

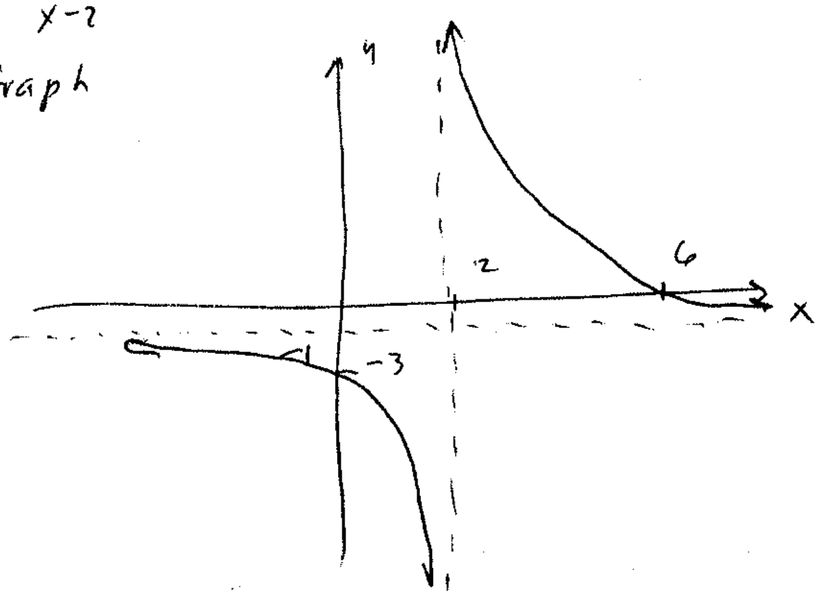
Sign Table

Factor		2		6	
$6-x$	$+$		$+$	0	$-$
$x-2$	$-$	0	$+$		$+$
$f(x)$	$-$	∞	$+$	0	$-$

From sign table

$\frac{2x}{x-2} < 3$ when $x \in (-\infty, 2) \cup (6, \infty)$ ans.

Graph



58. $\frac{x+1}{x+2} > \frac{x-3}{x+4}$ (15)

Solve to get everything on left side.

$$\frac{x+1}{\cancel{x}+2} - \frac{x-3}{\cancel{x}+4} > 0$$

$$\frac{(x+1)(x+4) - (x-3)(x+2)}{(x+2)(x+4)} > 0$$

$$\frac{x^2+5x+4-(x^2-x-6)}{(x+2)(x+4)} > 0$$

$$\frac{6x+10}{(x+2)(x+4)} > 0$$

vertical asymtotes $x = -4, -2$

x-intercept $= -5/3$, y intercept: $y = \frac{0+10}{(0+2)(0+4)} = \frac{5}{4}$

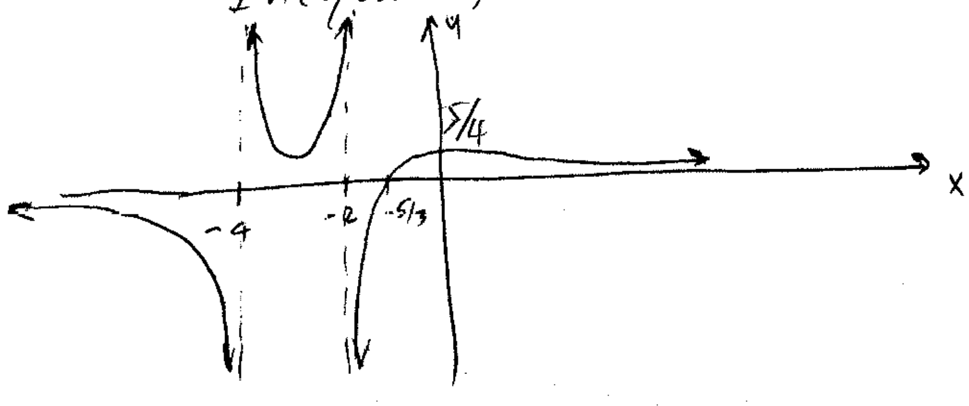
Horizontal asymptote $y = 0$

Sign Table

Factor		-4		-2		$-5/3$	
$6x+10$	$-$		$-$		$-$	0	$+$
$x+2$	$-$		$-$	∞	$+$		$+$
$x+4$	$-$	∞	$+$		$+$		$+$
$f(x)$	$-$	∞	$+$	∞	$-$	0	$+$

From sign table

Inequality holds when $x \in (-4,-2) \cup (-5/3, \infty)$ ans



(20) Appendix B3: 22, 26, 27, 28, 32, 50, 57, 58, 70, 74

[4] (22) $\left(\frac{1}{81}\right)^{1/4} = \frac{1^{1/4}}{81^{1/4}} = \frac{1}{3}$ ans

[4] (26) $64^{2/3} = \left(64^{1/3}\right)^2 = 4^2 = 16$ ans

[4] (27.) $(-32)^{1/5} = -32^{1/5} = -2$ ans

[4] (28.) $(-1/125)^{1/3} = -\frac{1^{1/3}}{125^{1/3}} = -\frac{1}{5}$ ans

[4] (32) $(121)^{-1/2} = \frac{1}{121^{1/2}} = \frac{1}{11}$ ans

50. $\frac{(2x^2+1)^{-6/5}(2x^2+1)^{6/5}(x^2+1)^{-1/5}}{(x^2+1)^{9/5}} = (2x^2+1)^{-6/5+6/5} \cdot (x^2+1)^{-1/5-9/5}$

$= (x^2+1)^{-10/5} = \frac{1}{(x^2+1)^2}$ ans

57. $\sqrt[4]{x^a}\,\sqrt[3]{x^b}\,\sqrt{x^{a/6}} = x^{a/4} \cdot x^{b/3} \cdot x^{a/12}$

$= x^{a/4 + a/12 + b/3} = x^{\frac{3a}{12} + \frac{a}{12} + \frac{b}{3}} = x^{4a/12 + b/3}$

$= x^{a/3 + \frac{b}{3}}$ ans or $x^{(a+b)/3}$ ans

58. $\sqrt[3]{27\sqrt{64x}} = \sqrt[3]{27 \cdot 8x^{1/2}} = \sqrt[3]{27 \cdot 8}\,\sqrt[3]{x^{1/2}} = 6x^{1/6}$ ans

70 $(x^2+y^2)^{1/2} \neq x+y$ Let $x=1,\ y=1$

$(1+1)^{1/2} \neq 1+1$ (answer vary)

$2^{1/2} \neq 2$

74. $x^{-1/2} \neq \frac{1}{x^2}$ let $x = 4$

$(4)^{-1/2} \stackrel{?}{=} \frac{1}{4^2} \Rightarrow \frac{1}{2} \neq \frac{1}{16}$ ✓

[12] Section 5.1: 4, 6, 8, 10

[3] 4 $\left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^2 = 2$ ans

[3] 6. $\left(3^{2+\sqrt{5}}\right)\left(3^{2-\sqrt{5}}\right) = 3^{2+\sqrt{5}+2-\sqrt{5}} = 3^4 = 81$ ans

[3] 8. $\frac{10^{\pi+2}}{10^{\pi-2}} = 10^{\pi+2-\pi+2} = 10^4 = 10{,}000$ ans

[3] 10. $\left[\left(\sqrt{3}\right)^{\pi}\right]^4 = \left[3^{\pi/2}\right]^4 = 3^{2\pi} = \left(3^2\right)^{\pi} = 9^{\pi}$ ans

[8] Find the inverse:

[4] 4 $f(x) = 3x^{-2/3}$

$y = 3x^{-2/3}$ Solve for x

$\frac{y}{3} = x^{-2/3}$

$\left(\frac{y}{3}\right)^{-3/2} = \left(x^{-2/3}\right)^{-3/2}$

$\left(\frac{y}{3}\right)^{-3/2} = x$

$\boxed{f^{-1}(x) = \left(\frac{x}{3}\right)^{-3/2}}$ ans.

Ck. $f(f^{-1}(x)) = f\left(\left(\frac{x}{3}\right)^{-3/2}\right)$

$= 3\left(\left(\frac{x}{3}\right)^{-3/2}\right)^{-2/3} = 3 \cdot \frac{x}{3} = x$

$g(x) = \frac{x^4}{81}$

$y = \frac{x^4}{81}$ Solve for x

$81y = x^4$

$(81y)^{1/4} = x$

$g^{-1}(x) = (81x)^{1/4} = 3x^{1/4}$ ans

ck. $g(g^{-1}(x)) =$

$g(3x^{1/4}) = \frac{(3x^{1/4})^4}{81}$

$= \frac{3^4 x}{81} = x$

[4] ⊛ $h(x) = 2(x+1)^{2\pi}$

$y = 2(x+1)^{2\pi}$ Solve for x

$\frac{y}{2} = (x+1)^{2\pi}$

$\left(\frac{y}{2}\right)^{\frac{1}{2\pi}} = x+1$

$x = \left(\frac{y}{2}\right)^{1/2\pi} - 1$

$\boxed{h^{-1}(x) = \left(\frac{x}{2}\right)^{\frac{1}{2\pi}} - 1}$ ans

ck $h(h^{-1}(x))$

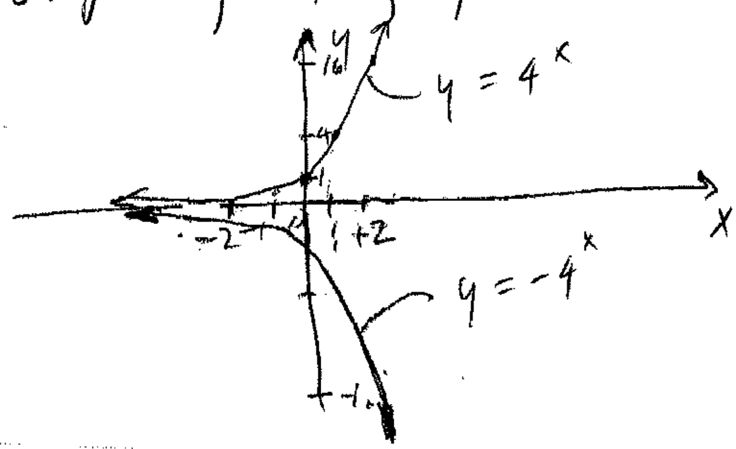
$h\left(\left(\frac{x}{2}\right)^{1/2\pi} - 1\right) =$

$2\left(\left(\frac{x}{2}\right)^{\frac{1}{2\pi}} - 1 + 1\right)^{2\pi} =$

$2\left(\left(\frac{x}{2}\right)^{1/2\pi}\right)^{2\pi} = 2\left(\frac{x}{2}\right) = x$

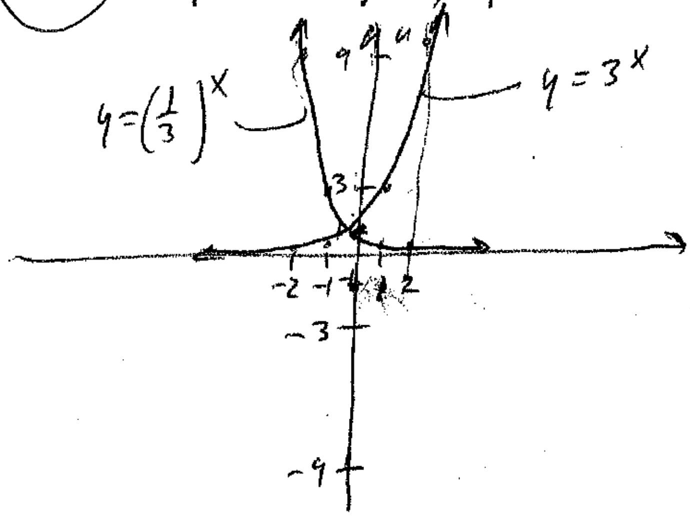
Section 5.1: 20, 22, 26, 32

20 Graph $y = 4^x$, $y = -4^x$



x	4^x	-4^x
0	1	-1
1	4	-4
2	16	-16
-1	1/4	-1/4
-2	1/16	-1/16

[5] 22. $y = \left(\frac{1}{3}\right)^x$, $y = 3^x$



(4)

x	$(1/3)^x$	3^x
0	1	1
1	1/3	3
2	1/9	9
-1	3	1/3
-2	9	1/9

[5] 26. $y = -3^x + 3$

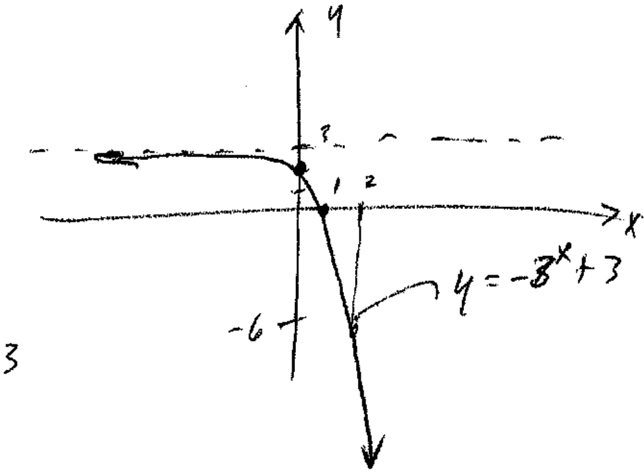
domain $= (-\infty, \infty)$

range $= (-\infty, 3)$

y intercept $= 2$

x intercept $= 1$

horizontal asymptote $y = 3$



32. $y = 1 - 3^{x-1}$

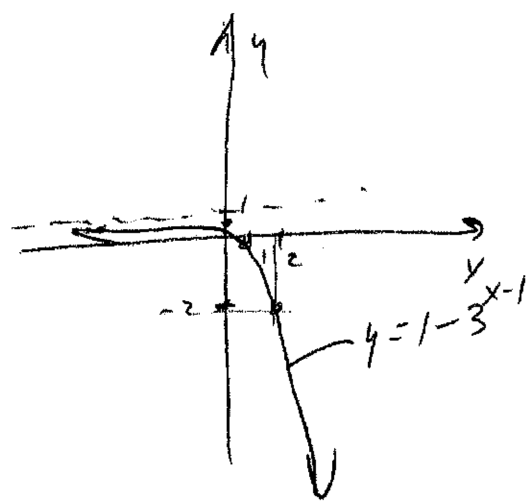
domain $= (-\infty, \infty)$

range $= (-\infty, 1)$

y intercept $= 2/3$ $\left(= 1 - \frac{1}{3}\right)$

x intercept $= 1$

horizontal asymptote $y = 1$



[12] Simplify

[4] $\dfrac{2^{x-1}}{4^{x+2}} = \dfrac{2^{x-1}}{(2^2)^{x+2}} = \dfrac{2^{x-1}}{2^{2x+4}} = 2^{x-1-2x-4}$

$= \boxed{2^{-x-5}} = \boxed{\dfrac{1}{2^{x+5}}}$ ans both ok.

[4] $\dfrac{3^{2x-1}}{27^{3-x}} = \dfrac{3^{2x-1}}{(3^3)^{3-x}} = \dfrac{3^{2x-1}}{3^{9-3x}} = 3^{2x-1-9+3x}$

$= \boxed{3^{5x-10}}$ ans

[4] $\dfrac{(5^{2x-1})^{1/2}}{625^x} = \dfrac{(5^{2x-1})^{1/2}}{(5^4)^x} = \dfrac{5^{x-1/2}}{5^{4x}} = 5^{x-1/2-4x}$

$= \boxed{5^{-3x-1/2}}$ ans

38. a) [8]

$V = \pi r^2 h = 1000 \qquad S = 2\pi r^2 + 2\pi r h = \dfrac{2\pi r^3 + 2\pi r^2 h}{r}$

Since $\pi r^2 h = 1000 \Rightarrow S(r) = \dfrac{2\pi r^3 + 2000}{r} \qquad (r > 0)$

Proof: [2]

X intercept

$$2\pi r^3 + 2000 = 0$$

$$r^3 = \frac{-1000}{\pi}$$

$$r = \frac{-10}{\sqrt[3]{\pi}} \quad \leftarrow [2]$$

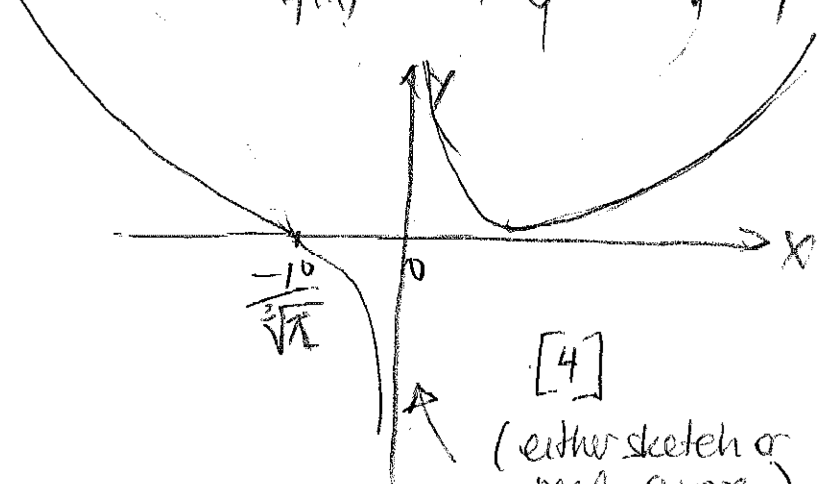
Y intercept None since $r \neq 0$

X asymptote $r = 0$

Y asymptote None.

as $x \to +\infty$ $y \to +\infty$ $x \to -\infty$, $y \to +\infty$

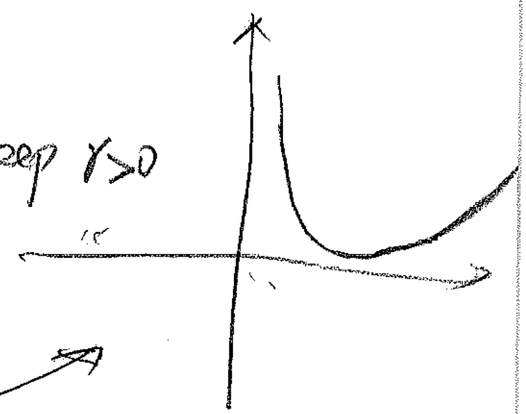
		$\frac{-10}{\sqrt[3]{\pi}}$		0	
$2\pi r^3 + 2000$	$-$	0	$+$		$+$
r	$-$		$-$	0	$+$
$f(x)$	$+$	0	$-$	0	$+$



[4]

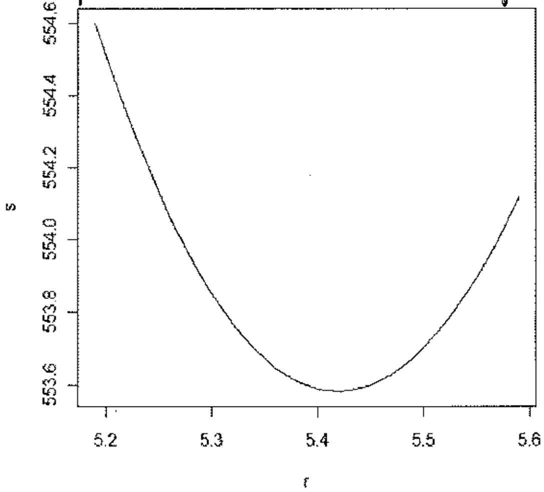
(either sketch or real curve.)

only keep $r > 0$ $\Rightarrow$



both ok.

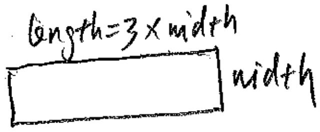
b) Using graphing utility to graph the surface area for the function obtained in part (a).



c) When the radius $r = 5.42$, the minimum surface area is obtained. The corresponding value for h is $\frac{V}{\pi r^2} = \frac{1000}{\pi r^2}$

$= 10.84$

35. For the first piece, the



first piece: width $= \frac{x}{2(2+1)}$

length $\frac{x}{2(2+1)} \times 2$

area: width × length $= \frac{x}{2(2+1)} \times \frac{x}{2(2+1)} \times 2$

Second piece: width $= \frac{16-x}{2\times(3+1)}$

length $\frac{16-x}{2\times(3+1)} \times 3$

area: $\frac{16-x}{2\times(3+1)} \times \frac{16-x}{2\times(3+1)} \times 3$

$\frac{x^2}{4(2+1)^2} \times 2 + \frac{(16-x)^2}{4(4\times4)} \times 3$

$= \frac{1}{4}\left[\frac{2x^2}{9} + \frac{3}{16}(x-16)^2\right] = \frac{1}{4}\left[\left(\frac{2}{9}+\frac{3}{16}\right)x^2 - 6x + 48\right]$

$= \frac{1}{4}\left[\frac{59}{144}x^2 - 6x + 48\right]$

if $X = \frac{-b}{2a} = \frac{-(-6)}{2\times\frac{59}{144}} = \frac{6}{\frac{2\times 59}{144}} = \frac{6\times 144}{2\times 59} = 7.32$

the total area of the two retangles reaches the minimum.