HOMEWORK 5 ANSWERS

16. $3x^3 - 48x = 0$

$3x(x^2 - 16) = 0$

$3x(x-4)(x+4) = 0$

$x_1 = 0$

$x_2 = 4$

$x_3 = -4$

18. $t - t^3 = 0$

$t(1 - t^2) = 0$

$t(1-t)(1+t) = 0$

$t_1 = 0$

$t_2 = 1$

$t_3 = -1$

26 $x^4 - 5x^2 + 6 = 0$

Let $t = x^2$

[10] $t^2 - 5t + 6 = 0$

$(t-2)(t-3) = 0$

$t_1 = 2 \quad t_2 = 3$ ← [5] for getting here

$\Rightarrow x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

$x^2 = 3 \Rightarrow x = \pm\sqrt{3}$

$\Rightarrow x = \{\pm\sqrt{2}, \pm\sqrt{3}\}$ ← [5] for correct solutions

28. $6y^2 = -5 - y^4$

Let $t = y^2$

$6t = -5 - t^2$

$t^2 + 6t + 5 = 0$

$(t+1)(t+5) = 0$

$t_1 = -1 \Rightarrow y^2 = -1 \Rightarrow y$ no solution

$t_2 = -5 \Rightarrow y^2 = -5 \Rightarrow y$ no solution

$y$ has no roots.

30 [10]

$5t^2 - 1 = 4t^4$

Let $w = t^2$

$5w - 1 = 4w^2$

$4w^2 - 5w + 1 = 0$

$(4w-1)(w-1) = 0$ $\rightarrow$ [5] for finding $t^2 = \frac{1}{4}$, $t^2 = 1$

$w_1 = \frac{1}{4}$ $t^2 = w_1 = \frac{1}{4} \Rightarrow t = \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}$

$w_2 = 1$ $t^2 = w_2 = 1 \Rightarrow t = \pm\sqrt{1} = \pm 1$

$\Rightarrow t$ has 4 roots

$\frac{1}{2}, -\frac{1}{2}, 1, -1$ $\leftarrow$ [5] for all solutions section 4.5: 2, 6, 10, 4

(2.) Find two numbers adding to 20 such that their squares are as small as possible.

[10]

$x$, $20-x$ are the numbers.

$y = x^2 + (20-x)^2$ ← we want to find minimum

$y = x^2 + 400 - 40x + x^2$

$y = 2x^2 - 40x + 400$ min occurs at $x = \frac{-b}{2a} = \frac{40}{4} = 10$

$x = 10$ $20 - x = 10$ the numbers are $\{10, 10\}$ ans

6. What is the largest possible area for a rectangle with perimeter 80 cm? (3)

Find an expression of x for other side.

$2x + 2y = 80$

$2y = 80 - 2x$

$y = 40 - x$

Then $A = xy = x(40-x) = 40x - x^2$

$x = -\frac{b}{2a} = \frac{-(40)}{2(-1)} = 20 \Rightarrow y = 20$

$A = 20^2 = 400\ cm^2$ ans

10. An objects height is given by

$h = 512t - 16t^2$ t in seconds, h in feet

Find maximum h and t when $h = 0$

$h = -16(t^2 - 32t)$ factor to simplify although not necessary

$t = \frac{-(-32)}{2(1)} = 16$

$h$ (at max) $= -16((16)^2 - 32(16))$

$h$ (at max) $= -16(-256) = 4096$ ft ans

t when $h = 0$, further factor

$h = -16t\underbrace{(t - 32)}_{0} = 0$ $t = 32$ sec. ans

4. State whether it makes sense to look at a highest or lowest pt. Find that point.

(a) $y = 2x^2 - 8x + 1$ $\quad a > 0$ there is a min at the vertex (lowest point)

$x = \frac{-b}{2a} = \frac{-(-8)}{2(2)} = \frac{8}{4} = 2$

$y = 2(2)^2 - 8(2) + 1 = 8 - 16 + 1 = -7$

The vertex is $(2, -7)$ ans

[10]

(b) $y = -3x^2 - 4x - 9$ $\quad a < 0$ there is a max at the vertex (highest point)

$x = \frac{-b}{2a} = \frac{-(-4)}{2(-3)} = -\frac{2}{3}$

Note: different methods work too (eg. complete square). [-3]/algebra error.

$y = -3\left(\frac{2}{3}\right)^2 - 4\left(-\frac{2}{3}\right) - 9 = -\frac{23}{3}$

The vertex is $\boxed{\left(-\frac{2}{3}, -\frac{23}{3}\right)}$ ans

(c) $h = -16t^2 + 256t$ $\quad a < 0$ there is a max at the vertex (highest point)

$t = \frac{-b}{2a} = \frac{-256}{2(-16)} = 8$

$h = -16(8)^2 + 256(8) = 1024$

The vertex is $(8, 1024)$ ans

[10]

(d) $f(x) = -(x+1)^2 + 1$ $\quad a < 0$, there is a max at the vertex (highest point)

Read by inspection

The vertex is $\boxed{(-1, 1)}$ ans

same comment as above

(e) $g(t) = t^2 + 1$ $\quad a > 0$, there is a min at the vertex (lowest point)

By inspection

The vertex is $(0, 1)$ ans

(f) $f(x) = 1000x^2 - x + 100$ $\quad a > 0$, there is a min at the vertex (lowest point)

$x = \frac{-b}{2a} = \frac{-(-1)}{2(1000)} = \frac{1}{2000}$

$f\left(\frac{1}{2000}\right) = 1000\left(\frac{1}{2000}\right)^2 - \frac{1}{2000} + 100 = 99.99975$ or $\frac{399999}{4000}$

The vertex is $\left(\frac{1}{2000}, 99.99975\right)$ ans

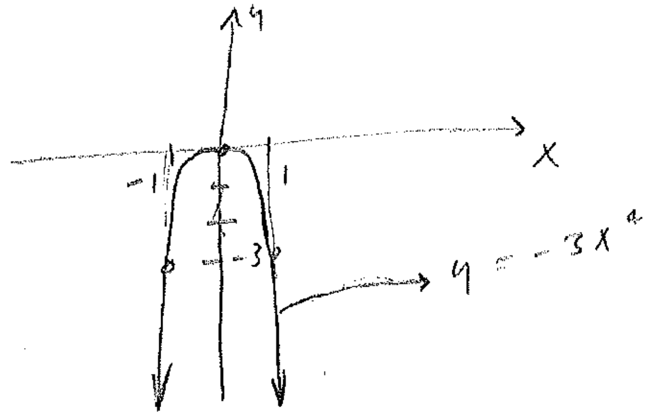
Section 4.6: 6, 10, 12

6. Sketch graph and specify x & y intercepts

$$y = -3x^4$$

y-intercept $y = -3(0)^4 = 0$ ans

x-intercept $y = 0 \Rightarrow x = 0$ ans




[10] 10. $y = -(x-4)^3 - 2$

[2] y-intercept $y = -(0-4)^3 - 2 = -(-4)^3 - 2$

$y = 64 - 2 = 62$ ans

[2] x-intercept $0 = -(x-4)^3 - 2$

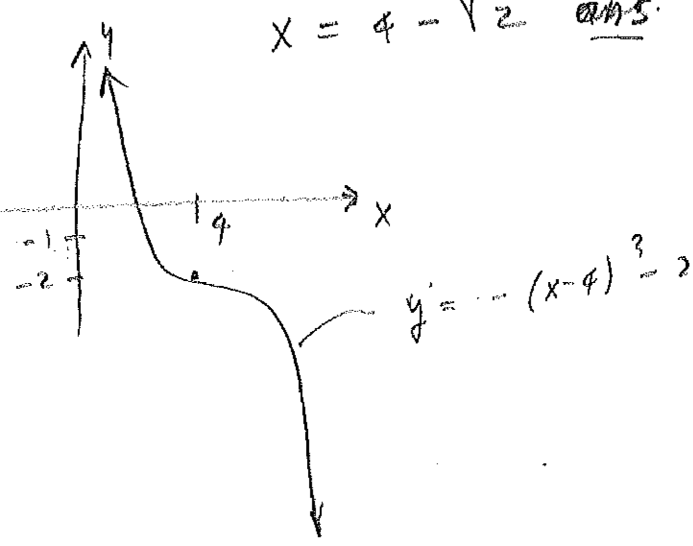
$(x-4)^3 = -2$

$x - 4 = \sqrt[3]{-2} = -\sqrt[3]{2}$

$x = 4 - \sqrt[3]{2}$ ans

[6]

(correct inflexion pt, correct shape needed).



12. $y = -2x^4 + 5$

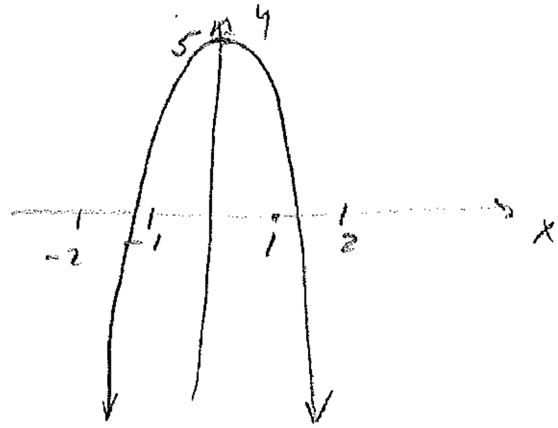
y-intercept: $y = -2(0)^4 + 5 = 5$ <u>ans</u>

x-intercepts $0 = -2x^4 + 5$

$\Rightarrow 2x^4 = 5 \Rightarrow x^4 = 5/2$

$x = \pm\sqrt[4]{5/2}$ <u>ans</u>

$\approx \pm 1.26$



Section 4.6: 28, 30, 32, 34, 36, 38, 40

28. $y = (x-3)(x-2)(x+1)$

a) It is fully factored

b) y-intercept $y = (-3)(-2)(1) = 6$ ans

x-intercepts: $\{-1, 2, 3\}$ ans

c)

| Factor | | $-1$ | | $2$ | | $3$ | | $\to x$ |
|---|---|---|---|---|---|---|---|---|
| $x-3$ | $-$ | | $-$ | | $-$ | $0$ | $+$ | |
| $x-2$ | $-$ | | $-$ | $0$ | $+$ | | $+$ | |
| $x+1$ | $-$ | $0$ | $+$ | | $+$ | | $+$ | |
| $f(x) = y$ | $-$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ | |

d) At $x = -1$ $y \approx (-1-3)(-1-2)(x+1)$

$y \approx 12(x+1) = 12x + 12$ linear ans

At $x = 2$ $y \approx (2-3)(2+1)(x-2)$

$y \approx -3(x-2) = -3x + 6$ linear ans

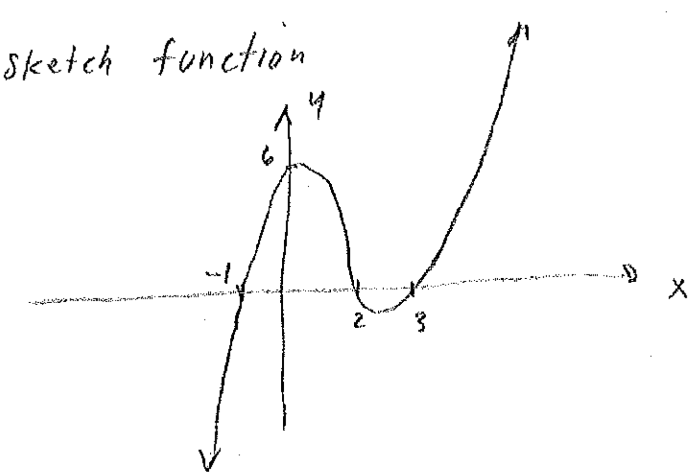
At $x = 3$ $y \approx (x-3)(3-2)(3+1)$

$y \approx 4(x-3) = 4x - 12$ linear ans

e) $f(x) \to -\infty$ as $x \to -\infty$ ans

$f(x) \to \infty$ as $x \to \infty$ ans

g) sketch function



30. $y = (x-3)(x-2)(x+2)$

[20] [2] a) It is fully factored

[4] b) y-intercept $y = (-3)(-2)(2) = 12$ ans.

x-intercepts: $\{-2, 2, 3\}$ ans (set factors to 0)

[4] c)

| Factor | | −2 | | 2 | | 3 | |
|---|---|---|---|---|---|---|---|
| $x-3$ | − | | − | | − | 0 | + |
| $x-2$ | − | | − | 0 | + | | + |
| $x+2$ | − | 0 | + | | + | | + |
| $y$ | − | 0 | + | 0 | − | 0 | + |

[4] d) At $x=-2$ $\quad y \approx (-2-3)(-2-2)(x+2)$

$y \approx 20(x+2) = 20x+40$ linear ans

At $x = 2$ $\quad y \approx (2-3)(x-2)(2+2)$

$y \approx -4(x-2) = -4x+8$ linear ans

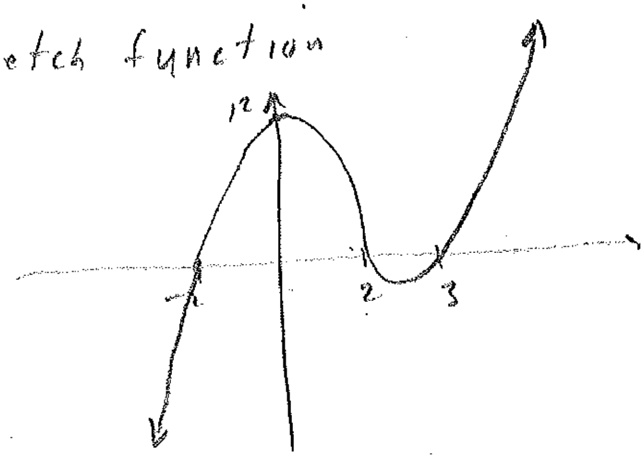
At $x = 3$ $\quad y \approx (x-3)(3-2)(3+2)$

$y \approx 5(x-3) = 5x-15$ linear ans

[2] e) $f(x) \longrightarrow -\infty$ as $x \to -\infty$ ans

$f(x) \longrightarrow \infty$ as $x \to \infty$ ans

[4] g) Sketch function



32. $y = x^3 - 9x$

a) not fully factored (don't have to state, just factor)

$y = x(x^2 - 9) = x(x-3)(x+3)$ ans

b) y-intercept $y = 0^3 - 9(0) = 0$ ans

x-intercepts $\{-3, 0, 3\}$ ans (set factors to 0 & solve)

c)

| Factor | | $-3$ | | $0$ | | $3$ | |
|---|---|---|---|---|---|---|---|
| $x$ | $-$ | | $-$ | $0$ | $+$ | | $+$ |
| $x-3$ | $-$ | | $-$ | | $-$ | $0$ | $+$ |
| $x+3$ | $-$ | $0$ | $+$ | | $+$ | | $+$ |
| $y$ | $-$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ |

d) At $x = -3$ $y \approx (-3)(-3-3)(x+3)$

$y \approx 18(x+3) = 18x + 54$ linear ans

At $x = 0$ $y \approx x(0-3)(0+3)$

$y \approx -9x$ linear ans

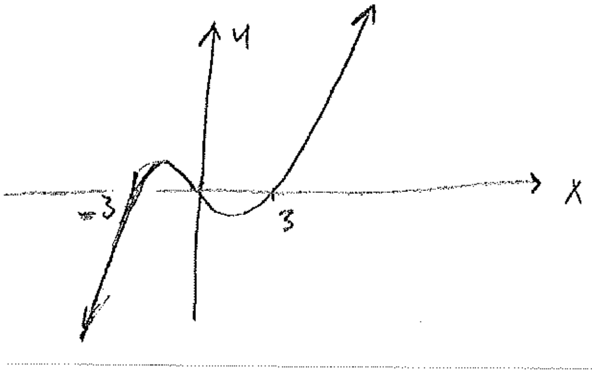
At $x = 3$ $y \approx (3)(x-3)(3+3)$

$y = 18(x-3) = 18x - 54$ linear ans

e) $f(x) \to -\infty$ as $x \to \infty$ ans

$f(x) \to \infty$ as $x \to \infty$ ans

g) Sketch



39. $y = x^3 - 5x^2 - x + 5$ (9)

[20] [2] a) Factor by grouping

$y = x^2(x-5) - (x-5)$

$y = (x-5)(x^2-1) = (x-5)(x-1)(x+1)$ ans

[4] b) y-intercept $y = 0^3 - 5(0)^2 - 0 + 5 = 5$ ans

x-intercepts $\{-1, 1, 5\}$ ans

[4] c)

| Factor | | $-1$ | | $1$ | | $5$ | |
|---|---|---|---|---|---|---|---|
| $x+1$ | $-$ | $0$ | $+$ | | $+$ | | $+$ |
| $x-1$ | $-$ | | $-$ | $0$ | $+$ | | $+$ |
| $x-5$ | $-$ | | $-$ | | $-$ | $0$ | $+$ |
| $y$ | $-$ | $0$ | $+$ | $0$ | $-$ | $0$ | $+$ |

[4] d) At $x = -1$ $\quad y \approx (-1-5)(-1-1)(x+1)$

$y \approx (-6)(-2)(x+1)$

$y \approx 12(x+1) = 12x + 12$ linear ans

At $x = 1$ $\quad y \approx (1-5)(x-1)(1+1)$

$y \approx -8(x-1) = -8x + 8$ linear ans

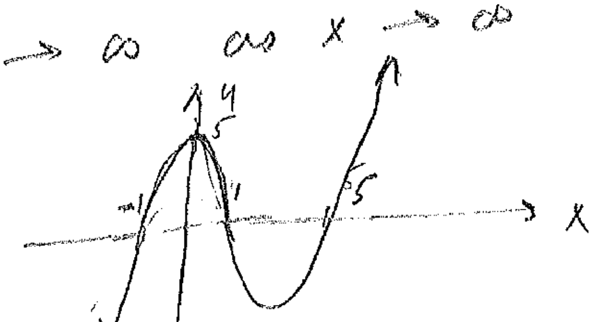
At $x = 5$ $\quad y \approx (x-5)(5-1)(5+1)$

$y \approx 24(x-5) = 24x - 120$ linear ans

[2] e) $f(x) \to -\infty$ as $x \to -\infty$ ans

$f(x) \to \infty$ as $x \to \infty$ ans

[4] g) Sketch



36. $y = (x-1)(x-4)^2$

a) It is full factored ans

b) y-intercept $y = (0-1)(0-4)^2 = -16$ ans

x-intercept $\{1, 4\}$ ans

c) Sign table

| Factor | | 1 | | 4 | |
|---|---|---|---|---|---|
| $x-1$ | $-$ | 0 | $+$ | | $+$ |
| $(x-4)^2$ | $+$ | | $+$ | 0 | $+$ |
| $y$ | $-$ | 0 | $+$ | 0 | $+$ |

d) At $x = 1$ $y \approx (x-1)(1-4)^2$

$y \approx 9(x-1) = 9x - 9$ linear ans

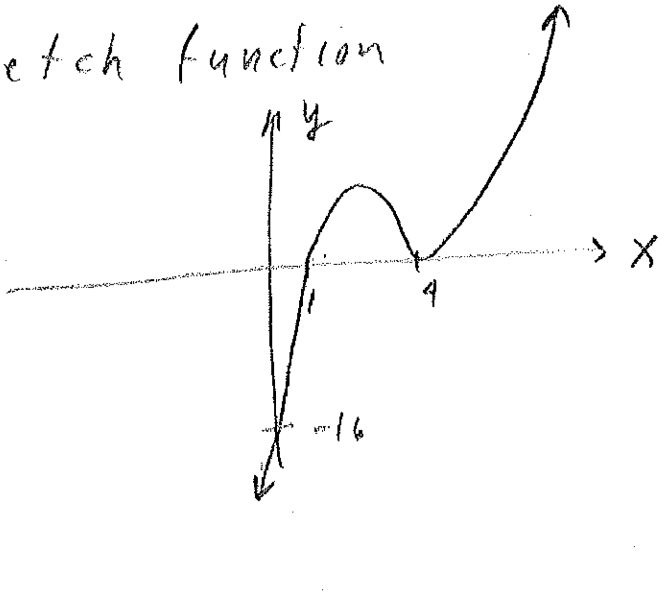
At $x = 4$ $y = (4-1)(x-4)^2$

$y = 3(x-4)^2$ parabolic (local min) ans

e) $f(x) \to -\infty$ as $x \to -\infty$ ans

$f(x) \to \infty$ as $x \to \infty$ ans

g) Sketch function



38 $y = (x-1)^2(x-4)^2$

a) Fully factored ans

b) y-intercept : $y = (0-1)^2(0-4)^2 = 16$ ans

x-intercepts $\{1, 4\}$ ans

c)

| Factor | | 1 | | 4 | |
|---|---|---|---|---|---|
| $(x-1)^2$ | + | 0 | + | | + |
| $(x-4)^2$ | + | | + | 0 | + |
| $y$ | + | 0 | + | 0 | + |

d) At $x = 1$ $y \approx (x-1)^2(x-4)^2$

$y \approx 9(x-1)^2$ parabolic ans (local min at $x=1$)

At $x = 4$ $y \approx (4-1)^2(x-4)^2$

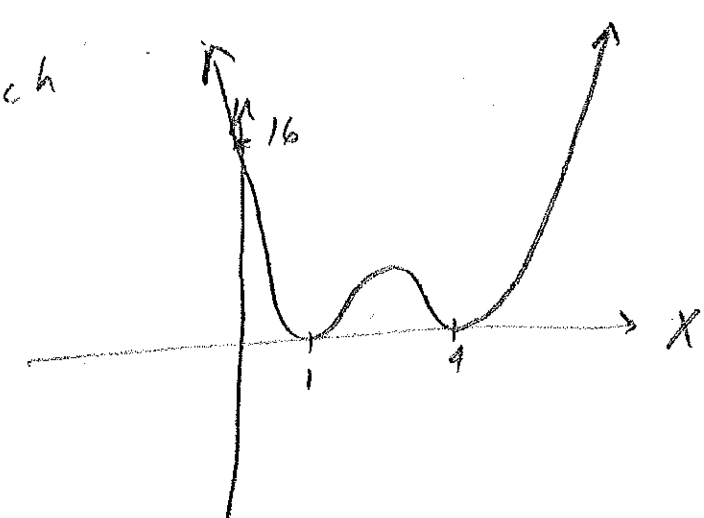
$y = 9(x-4)^2$ parabolic ans (local min at $x=4$)

note: comment "local min at x" not required for grading

e) $f(x) \to \infty$ as $x \to -\infty$ ans

$f(x) \to \infty$ as $x \to \infty$ ans

g) Sketch



40 $y = x^2(x-4)(x+2)$

a) full factored ans

b) y-intercept $y = 0^2(0-4)(0+2) = 0$ ans

x-intercepts $\{-2, 0, 4\}$ ans

c) Factor

| Factor | $x<-2$ | $-2$ | $-2<x<0$ | $0$ | $0<x<4$ | $4$ | $x>4$ |
|---|---|---|---|---|---|---|---|
| $x^2$ | + | | + | 0 | + | | + |
| $x-4$ | − | | − | | − | 0 | + |
| $x+2$ | − | 0 | + | | + | | + |
| $y$ | + | 0 | − | 0 | − | 0 | + |

d) At $x = -2$ $\quad y \approx (-2)^2(-2-4)(x+2)$

$y \approx -24(x+2) = -24x - 48$ linear ans

At $x = 0$ $\quad y \approx x^2(0-4)(0+2)$

$y \approx -8x^2$ parabolic ans (local max at $x=0$)

At $x = 4$ $\quad y \approx 4^2(x-4)(4+2)$

$y \approx 96(x-4) = 96x - 384$ linear ans

e) $f(x) \to \infty$ as $x \to -\infty$ ans

$f(x) \to \infty$ as $x \to \infty$ ans

g) Sketch