HOMEWORK 4 ANSWERS.

App.B4. Review of factoring: 2, 4, 6, 10, 16, 26, 62.

[8] 2. a) $1-t^4=(1+t^2)(1-t^2)=(1+t^2)(1+t)(1-t)$ **<-Ans** [4] (-2 if don't factor second term)

b) $x^6+x^5+x^4=x^4(x^2+x+1)$ **<-Ans** [4] NP

c) $u^2v^2-225=(uv+15)(uv-15)$ **<-Ans**

d) $81x^4-x^2=x^2(81x^2-1)=x^2(9x+1)(9x-1)$

4. Factor by trial and error:

a) $2x^2-7x+4=(2x-?)(x-?)$ Possible integer factors of 4 are [2,2] or [4,1]. None work. Irreducible over the integers. **<-Ans**

Possible irrational factors are: $(2x-\frac{7+\sqrt{17}}{2})(x-\frac{7-\sqrt{17}}{4})$

b) $2x^2+7x-4=(2x-1)(x+4)$ **<-Ans**

c) $2x^2+7x+4=(2x+?)(x+?)$ Irreducible over the integers **<-Ans**

Possible irrational factors are: $(2x+\frac{7+\sqrt{17}}{2})(x+\frac{7-\sqrt{17}}{4})$

d) $-2x^2-7x+4=(-1)(2x^2+7x-4)=-(2x-1)(x+4)$ *or* $(1-2x)(x+4)$ **<-Ans**

6. Factor by grouping.

a) $x^4-2x^3+3x-6=x^3(x-2)+3(x-2)=(x-2)(x^3+3)$ **<-Ans**

b) $a^2x+bx-a^2z-bz=x(a^2+b)-z(a^2+b)=(a^2+b)(x-z)$ **<-Ans**

Note: If a pair starts with a negative term, always factor out a negative.

[12] 10. a) $x^4-x^2=x^2(x^2-1)=x^2(x+1)(x-1)$ **<-Ans** [4] (-2 if don't factor second term)

b) $3x^4-48x^2=3x^2(x^2-16)=3x^2(x+4)(x-4)$ **<-Ans** [4] (-2 if don't factor second term)

c) $3(x+h)^4-48(x+h)^2=3(x+h)^2((x+h)^2-16)=3(x+h)^2(x+h+4)(x+h-4)$ **<-Ans** [4]

(-2 if don't factor second term)

14 (a) $x^2 - x + 6$

$\Delta = B^2 - 4AC$

$A = 1$

$B = -1$

$C = 6$

$\Delta = (-1)^2 - 4 \times 1 \times 6$

$= 1 - 24$

$= -23$

$\Delta < 0$

so this polynomial is irreducible (cannot be factored).

(b) $x^2 + x - 6$

try and error

$1 \quad 3$

$1 \quad -2$

$= (x+3)(x-2)$

[10]

22 (a) $x^3 + x^2 + x$

$= x(x^2 + x + 1)$ [5]

(b) $x^3 + 2x^2 + x$

$= x(x^2 + 2x + 1)$

$= x(x+1)^2$ [5] (-2 if don't factor second term)

24. (a) $3(x+5)^3+2(x+5)^2$

$= (x+5)^2[3(x+5)+2]$

$=(x+5)^2[3x+15+2]$

$=(x+5)^2(3x+17)$

(b) $a(x+5)^3+b(x+5)^2$

$=(x+5)^2[a(x+5)+b]$

$=(x+5)^2[ax+5a+b]$

30. x^2+64

Irreducible, cannot be factored.

Complete the square

2. 1/ x^2+2x

$=(x+1)^2-1$

Vertex $(-1,-1)$

2/ $-x^2+3x$

$=-[x^2-3x]$

$=-[x^2-3x+\frac{9}{4}-\frac{9}{4}]$

$=-(x-\frac{3}{2})^2+\frac{9}{4}$

vertex $(\frac{3}{2},\frac{9}{4})$

[15] (3) $2x^2-2x+1$

$= 2(x^2-x+\frac{1}{2})$

$= 2(x^2-x+\frac{1}{4}-\frac{1}{4}+\frac{1}{2})$

$= 2\left[(x-\frac{1}{2})^2+\frac{1}{4}\right] = 2\cdot(x-\frac{1}{2})^2+\frac{1}{2}$ [10] (−4/ algebra error)

vertex $(\frac{1}{2}, \frac{1}{2})$ [5]

(4) $-3x^2+x-1$

$= -3(x^2-\frac{1}{3}x+\frac{1}{3})$

$= -3(x^2-\frac{1}{3}x+\frac{1}{36}-\frac{1}{36}+\frac{1}{3})$

$= -3\{(x-\frac{1}{6})^2+\frac{11}{36}\}$

$= -3(x-\frac{1}{6})^2-\frac{11}{12}$

vertex $(\frac{1}{6}, -\frac{11}{12})$

[15] (5) hx^2+ax

$= h(x^2+\frac{a}{h}x)$

$= h\left[x^2+2x\frac{a}{2h}x+(\frac{a}{2h})^2-(\frac{a}{2h})^2\right]$

$= h\cdot(x+\frac{a}{2h})^2-h\,\frac{a^2}{4h^2}$

$= h(x+\frac{a}{2h})^2-\frac{a^2}{4h}$ [10]

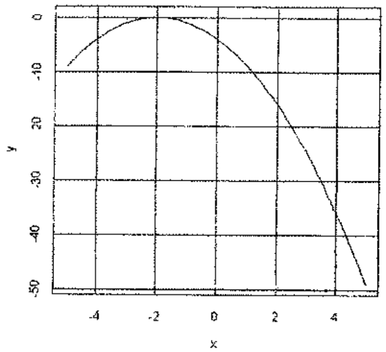
vertex $(-\frac{a}{2h}, -\frac{a^2}{4h})$ [5] (−4/ algebra error).

6. Function : $y = -(x+2)^2$ i) discriminant $\Delta = 0$

(ii) y-intercept first: $y = -(0 + 2)^2 = -4$ **<-Ans**
x-intercept(s): $0 = -(x+2)^2 \Rightarrow x = -2$ **<-Ans**

(iii) It is in vertex form: The vertex is (-2,0) **<-Ans**
Note: After completing the square to get $y = a(x-h)^2 + k$, the vertex is known to be (h,k).

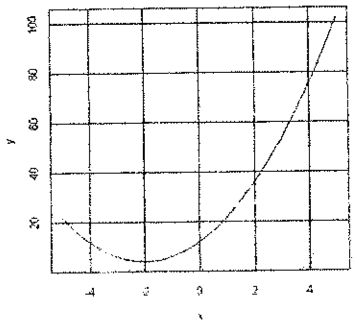
iv) Graph below. Confirms our analysis.



8. Function : $y = 2(x+2)^2 + 4 = 2(x^2+4x+4)+4 = 2x^2+8x+12$ (i) discriminant $\Delta = 8^2 - 4\times 2\times$ $= 64 - 96$ $= -32$
(ii) y-intercept first: $y = 2(0 + 2)^2 + 4 = 12$ **<-Ans**
x-intercept(s): $0 = 2(x+2)^2 + 4 \Rightarrow 0 = 2x^2 + 8x + 8 + 4 \Rightarrow$
$0 = 2x^2 + 8x + 12 \Rightarrow 0 = x^2 + 4x + 6$
Compute descriminant: $4^2 - 4(1)(6) = -8 < 0$
No real x intercepts **<-Ans**

(iii) It is in vertex form: The vertex is (-2,4) **<-Ans**
Note: After completing the square to get $y = a(x-h)^2 + k$, the vertex is known to be (h,k).

(iv) Graph below. Confirms our analysis.



[20]

10. [5] Function : $y = x^2 + 6x - 1$ (i) discriminant $\Delta = 6^2 - 4\times(-1) = 36+4 = 40$
[5] (ii) y-intercept first: $y = 0^2 + 6(0) - 1 = -1$ **<-Ans**
x-intercept(s): Use quadratic formula (see next page)

[1] for y-intercept
[2] for each x-intercept
(see next page)

$$x=\frac{-6\pm\sqrt{36-4(1)(-1)}}{2}=\frac{-6\pm\sqrt{40}}{2}=-3\pm\sqrt{10}$$ **<-Ans**

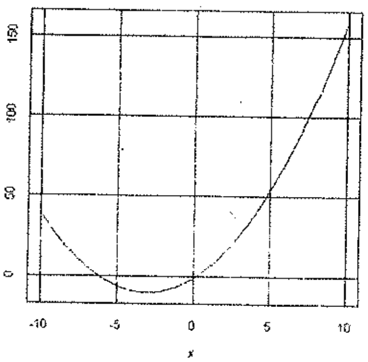
[5] (iii) It is not in vertex form. We need to complete the square

$$y=x^2+6x-1=(x^2+6x+9)-9-1=(x+3)^2-10$$

The vertex is (-3,-10) **<-Ans**

[3] for complete square
[2] for vertex OR
[2] for $x_v=-\frac{b}{2a}$
[3] for y_v

[5] (iv) Graph below. Confirms our analysis.



Make sure
axes are labeled
x-int are in right place
y-int is in right place
vertex is in right place.

[20]

(5) 12. Function : F(x) = x^2 - 3x + 4 (i) discriminant $\Delta=(-3)^2-4\times4=9-16=-7$

[5] (ii) y-intercept first: y = 0^2 - 3(0) + 4 = 4 **<-Ans**

x-intercept(s): Use discriminant: $(-3)^2$ -4(1)(4) = 9 - 16 = -7

No real x intercepts **<-Ans**

[3] for y-intercept
[2] for saying "no x-intercept"

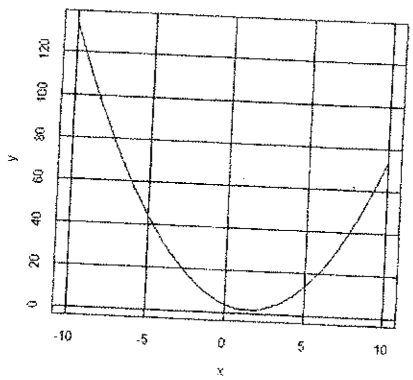
[5] (iii) It is not in vertex form. We need to complete the square

$$F(x)=x^2-3x+4=(x^2-3x+9/4)-9/4+4=(x-3/2)^2+7/4$$

The vertex is (3/2,7/4) **<-Ans**

[3] for correct complete square
[2] for vertex.
OR if they use $x_v=-\frac{b}{2a}$, [2] for this and [3] for [y_v].

[5] (iv) Graph below. Confirms our analysis. [5]



Make sure axes are labeled,
y-intercept is in right place
x-intercept are " " " (i.e none)
vertex is in right place.

16. Function : y = $2x^2$ + 3x - 2 (i) discriminant $\Delta=3^2-4\times2\times(-2)=9+16=25$

(ii) y-intercept first: y = $2(0)^2$ + 3(0) - 2 = -2 **<-Ans**

x-intercept(s): Use quadratic formula

$$x=\frac{-3\pm\sqrt{9-4(2)(-2)}}{4}=\frac{-3\pm\sqrt{25}}{4}=\frac{-3\pm5}{4}=-2\quad or\quad 1/2$$

x intercepts are {-2, 1/2} **<-Ans**

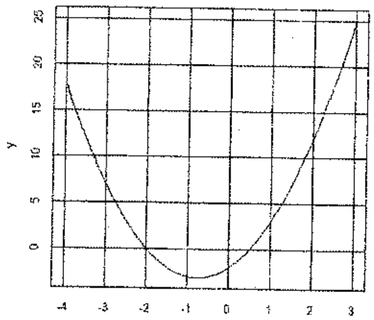
Note: Factoring could have been used here.

(iii) It is not in vertex form. We need to complete the square

$$y=2x^2+3x-2=2(x^2+\frac{3}{2}x+9/16)-18/16-2=2(x+3/4)^2-50/16$$

The vertex is (-3/4,-50/16) **<-Ans**

(iv) Graph below. Confirms our analysis.



18. Function : $y = -3x^2 + 12x$ (i) discriminant: $\Delta = 12^2 - 4 \times (-3) \times 0 = 144$

(ii) y-intercept first: $y = -3(0)^2 + 12(0) = 0$ **<-Ans**

x-intercept(s): Use factoring: $0 = -3x(x - 4)$

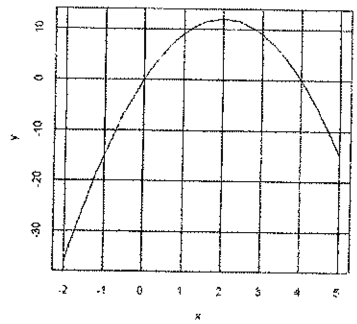
x intercepts are {0, 4} **<-Ans**

(iii) It is not in vertex form. We need to complete the square

$$y = -3x^2 + 12x = -3(x^2 - 4x + 4) + 12 = -3(x-2)^2 + 12$$

The vertex is (2,12) **<-Ans**

(iv) Graph below. Confirms our analysis.



20. Function : $s = -(1/4)t^2 + t - 1$ (i) discriminant $\Delta = 1 - 4 \times (-1) \times (-\frac{1}{4}) = 0$

(ii) y-intercept first: $s = -(1/4)(0)^2 + (0) - 1 = -1$ **<-Ans**

x-intercept(s): Use quadratic formula:

$$t = \frac{-1 \pm \sqrt{1 - 4(-1/4)(-1)}}{2(-1/4)} = \frac{-1}{-(1/2)} = 2$$

'x' intercept is 2 **<-Ans**

Note: Factoring could be used here.

(iii) It is not in vertex form. We need to complete the square

$$s = -(1/4)t^2 + t - 1 = -(1/4)(t^2 - 4t + 4) + 1 - 1 = -(1/4)(t-2)^2$$

The vertex is (2,0) **<-Ans**

(iv) Graph below. Confirms our analysis.