HW3 ANSWERS

Section 3.6: 4, 9, 14, 18, 36, 50

[12] 4 Find which pair of functions are inverses. We will use the rule: f(g(x)) = x or g(f(x)) = x when functions are inverses.

[3] (a) f(x) = -3x + 2; g(x) = 2/3 - (1/3)x

f(g(x)) = -3(2/3 - (1/3)x) + 2 = -2 + x + 2 = x

These are inverses. **<-Ans**

[3] (b) F(x) = 2x + 1; G(x) = (1/2)x - 1

F(G(x)) = F((1/2)x - 1) = 2(1/2)x - 1) + 1 = x - 2 + 1 = -1

These are not inverses. **<-Ans**

[3] (c) G(x) = x^3; H(x) = 1 - x^3

G(H(x)) = $(1 - x^3)^3 \neq x$

These are not inverses. **<-Ans**

[3] (d) f(t) = t^3; g(t) = $\sqrt[3]{t}$

f(g(t)) = $(\sqrt[3]{t})^3 = x$

These are inverses. **<-Ans**

Note: One should usually check the other way, also.

9. Let f(x) = x^3+2+1. Assume f^{-1} exist. Evaluate the following:

(a) $f[f^{-1}(4)]=4$ *since* $f(f^{-1}(x))=x$ **<-Ans**

(b) $f^{-1}[f(-1)]=-1$ *since* $f^{-1}(f(x))=x$ **<-Ans**

(c) $(f\circ f^{-1}(\sqrt{2})=\sqrt{2}$ *since* $(f\circ f^{-1}(x)=f(f^{-1}(x))=x$ **<-Ans**

(d) $f[f^{-1}(t+1)]=t+1$ *since* $f(f^{-1}(x))=x$ **<-Ans**

[15] 14. Let f(x) = (1/3)x - 2.

(a) Find $f^{-1}(x)$. Rewrite in terms of x and y. Swap x and y. Solve for y. Then rename in function notation. [5]

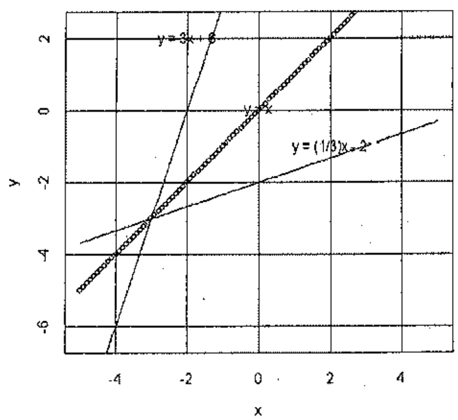
$$y=\frac{1}{3}x-2 \quad => \quad x=\frac{1}{3}y-2 \quad => \quad \frac{1}{3}y=x+2 \quad => \quad y=3x+6 \quad => \quad f^{-1}(x)=3x+6$$ **<-Ans**

[5] (b) Verify.

$$f(f^{-1}(x)) = f(3x+6) = \frac{1}{3}(3x+6) - 2 = x + 2 - 2 = x$$

$$f^{-1}(f(x)) = f(\frac{1}{3}x - 2) = 3(\frac{1}{3}x - 2) = 6 = x - 6 + 6 = x$$

[5] (c) Sketch to show symmetry. The lower line is for f(x) = (1/3)x - 2. The line with the greater slope is for the inverse, $f^{-1}(x) = 3x + 6$. The line in dots represents the line y = x. Note the symmetry about y = x



18. Let f(x) = (2x-3)/(x+4). Find $f^{-1}(x)$ and the domain and range of f and f^{-1}. Again, writing the equation with y, swapping x and y, solving for y gives the inverse function.

$$y = \frac{2x-3}{x+4} \quad \Rightarrow \quad x = \frac{2y-3}{y+4} \quad \Rightarrow \quad x(y+4) = 2y - 3 \quad \Rightarrow$$

$$xy - 2y = -4x - 3 \quad \Rightarrow \quad y(x-2) = -4x - 3 \quad \Rightarrow \quad y = \frac{4x+3}{2-x}$$

$$f^{-1}(x) = \frac{4x+3}{2-x}$$ **<-Ans**

To get the domain of f(x), notice it is defined for all x values except x=-4. Hence, Domain f = $(-\infty,-4)\cup(-4,\infty)$ **<-Ans** What about the range? Its is helpful to graph the function to get the range. Graphing in the standard window gives you the idea that the range covers all values except possibly y = 2. In fact, that is the horizontal asymptote of the function, to be discussed later. To be sure, let f(x) = 2 and solve for x:

$$2 = \frac{2x-3}{x+4} \quad \Rightarrow \quad 2x + 8 = 2x - 3 \quad \Rightarrow \quad 8 = -3 \quad \textit{no solution}$$

So for the range: Range f = $(-\infty,2)\cup(2,\infty)$ **<-Ans**

The domain of f^{-1} can be seen to be all values except where x = 2. This also confirms our previous work.

Domain $f^{-1} = (-\infty,2)\cup(2,\infty)$ **<-Ans**

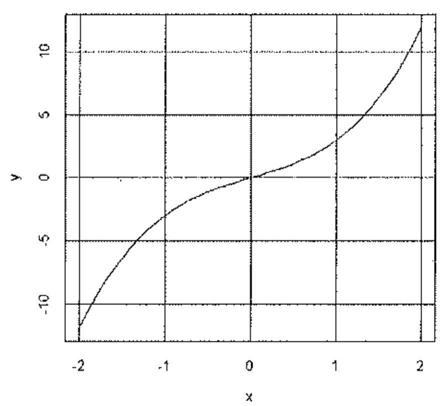
In a like manner one can show that $f^{-1}(x)$ is never -4.

Range $f^{-1} = (-\infty,-4)\cup(-4,\infty)$ **<-Ans**

Notice that the range of f is the domain of f^{-1} and vice versa. This confirms our analysis.

36. Use the graphing utility to show that the function $f(x)=x^3+2x$ passes the horizontal line test and is therefore one-to-one.

It passes the horizontal line test and is one-to-one. Graph below **<-Ans**



[20] 50. Investigate the inverse of a composite function.

(a) = Let $f(x)=2x+1$ *and* $g(x)=\frac{1}{4}x-3$

[2] i) $f(g(x))=f(\frac{1}{4}x-3)=2(\frac{1}{4}x-3)+1=\frac{1}{2}x-5$ **<-Ans**

[2] ii) $g(f(x))=g(2x+1)=\frac{1}{4}(2x+1)-3=\frac{1}{2}x+\frac{1}{4}-3=\frac{1}{2}x-\frac{11}{4}$ **<-Ans**

[2] iii) Find f^{-1}: $y=2x+1$ => $x=2y+1$ => $2y=x-1$ => $y=\frac{1}{2}x-\frac{1}{2}$

$f^{-1}(x)=\frac{1}{2}x-\frac{1}{2}$ **<-Ans**

[2] iv) Find $g^{-1}(x)$: $y=\frac{1}{4}x-3$ => $x=\frac{1}{4}y-3$ => $\frac{1}{4}y=x+3$ => $y=4x+12$

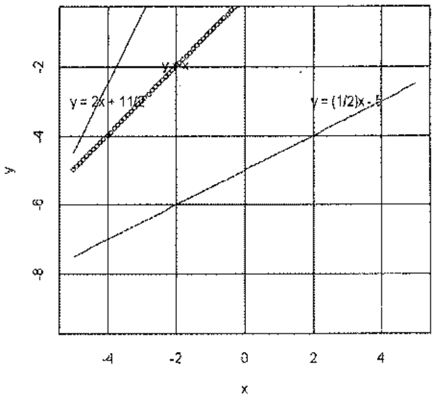
$g^{-1}(x)=4x+12$ **<-Ans** (See next page)

[2] v) $f^{-1}(g^{-1}(x)) = f^{-1}(4x+12) = \frac{1}{2}(4x+12) - \frac{1}{2} = 2x + 6 - \frac{1}{2} = 2x + \frac{11}{2}$ **<-Ans**

[2] vi) $g^{-1}(f^{-1}(x)) = g^{-1}(\frac{1}{2}x - \frac{1}{2}) = 4(\frac{1}{2}x - \frac{1}{2}) + 12 = 2x + 10$ **<-Ans**

(b) Graph the functions of i) and v) above an see if they are symmetric about y = x.
Not the best graph, but it does show that they are not symmetrical about y = x.

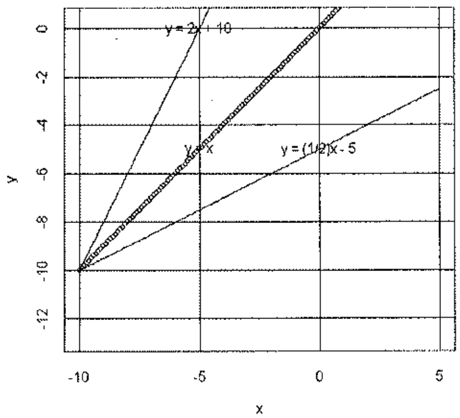
[4]



(c) Graph the functions of i) and vi) above an see if they are symmetric about y = x.

The graph below shows the relationship between the function and its inverse that we expect.

[4].



Sec.1.6. Equations of lines: 2(a), 4, 9, 10, 12, 18, 20, 21, 22, 26(graph not required), 32, 34, 36.

2(a) Compute the slope passing through (-3,0) and (4,9)

$$m = \frac{y2 - y1}{x2 - x1} = \frac{9-0}{4-(-3)} = \frac{9}{7}$$ **<-Ans**

[6] 4. Compute the slope of the line in the figure using each pair of points indicated.

[2] (a) A and B: $m = \frac{-2-0}{3-2} = \frac{-2}{1} = -2$ **<-Ans**

[2] (b) B and C: $m = \frac{0-6}{2-(-1)} = \frac{-6}{3} = -2$ **<-Ans**

[2] (a) A and C: $m = \frac{-2-6}{3-(-1)} = \frac{-8}{4} = -2$ **<-Ans**

9. Straight line graph given for cigarette production. Find Δt and ΔN for the period 1990->1993. (see later for solution)

Δt = 1993-1990 = 3 years **<-Ans**

ΔN = 5.3 - 5.419 = -0.119 trillion cigs. **<-Ans**

10. Compute the slope from 9.

$$\frac{\Delta N}{\Delta t} = \frac{-0.119}{3} =$$ -0.0397 trillion cigs./year **<-Ans**

[6] 12. Refer to figure in the book on page 53.

(a) List the slopes m1, m2, and m3 in increasing size: m3, m2, m1 **<-Ans** [3]

(b) List b1, b2, b3 n order of increasing size: b1, b2, b3 **<-Ans** [3]

18. Find the equation of the line passing through the points given.

(a) (7,9) and (-11,9):

$$m = \frac{9-9}{-11-7} = 0$$ Horizontal line: y = 9 **<-Ans**

(b) (5/4,2) and (3/4,3):

$$m=\frac{3-2}{3/4-5/4}=\frac{1}{-2/4}=-2$$
$$y-y1=m(x-x1)$$
$$y-3=-2(x-3/4)$$
$$y-3=-2x+3/2$$
$$y=-2x+9/2$$ **<-Ans**

Note: You can use any point as (x1,y1) for point-slope formula.

(c) (12,13) and (13,12):

$$m=\frac{12-13}{13-12}=\frac{-1}{1}=-$$
$$y-y1=m(x-x1)$$
$$y-12=-1(x-13)$$
$$y-12=-x+13$$
$$y=-x+25$$ **<-Ans**

Note: You can use any point as (x1,y1) for point-slope formula.

[4] 20. Write the equations of the horizontal and vertical lines passing through point (5,8)

Vertical line has x coordinate 5: x = 5 **<-Ans** [2]
Horizontal line has y coordinate 8: y = 8 **<-Ans** [2]

21. The graph of the line x = 0 is on the y-axis. **<-Ans**

22. The graph of the line y = 0 is on the x-axis. **<-Ans**

25 26 Find the equation of the lines described in y = mx + b form.
(a) Passes through (-7,-2) and (0,0)

[5]

$$m=\frac{0-(-2)}{0-(-7)}=\frac{2}{7}$$
$$y-y1=m(x-x1)$$
$$y-0=(2/7)(x-0)$$
$$y=(2/7)x$$ **<-Ans**

[5] (b) Passes through (6,-3) and has y-intercept 8

y-intercept = 8 is the point (0,8)

$$m=\frac{8-(-3)}{0-6}=-\frac{11}{6}$$
$$y-y1=m(x-x1)$$
$$y-8=-(11/6)(x-0)$$
$$y=-(11/6)x+8$$ **<-Ans** See next page for (c)-(e)

[5] (c) Passes through (0,-1) and has the same slope as 3x + 4y = 12
Solve the equation for the slope: y = -(3/4)x + 3 => m = -3/4

$$y - y1 = m(x - x1)$$
$$y - (-1) = -(3/4)(x - 0)$$
$$y = -(3/4)x - 1$$ **<-Ans**

[5] (d) Passes through (6,2) and has the same x-intercept as the line -2x + y = 1
Setting y to 0 to solve for the x-intercept: -2x = 1 => x = -1/2
The x-intercept is (-1/2,0)

$$m = \frac{0-2}{-1/2-6} = \frac{-2}{-(13/2)} = \frac{4}{13}$$
$$y - y1 = m(x - x1)$$
$$y - 2 = (4/13)(x - 6)$$
$$y = (4/13)x - 24/13 + 2$$
$$y = (4/13)x + 2/13$$ **<-Ans**

[5] (e) Has x-intercept of -6 and y-intercept of $\sqrt{2}$
These are the two points (-6,0) and (0,$\sqrt{2}$)

$$m = \frac{\sqrt{2}-0}{0-(-6)} = \frac{\sqrt{2}}{6}$$
$$y - y1 = m(x - x1)$$
$$y - 0 = (\sqrt{2}/6)(x - (-6))$$
$$y = (\sqrt{2}/6)x + 6$$ **<-Ans**

32. Are the lines y = x + 1 and y = 1 - x parallel, perpendicular, or neither.

Slope of first: m1 = 1; Slope of second: m2 = -1.
Since m1 = -1/m2 [1 = -1/(-1) = 1] the lines are perpendicular. **<-Ans**

34. Find the equations (slope-intercept and standard form) of the line parallel to 4x + 5y = 20 which passes through (0,0).

Get the slope of the line 4x + 5y = 20 by solving for y:
5y = -4x + 20 => y = -(4/5)x + 4 Slope = -(4/5)

$$y - y1 = m(x - x1)$$
$$y - 0 = -(4/5)(x - 0))$$
$$y = -(4/5)x \quad \textit{and} \quad 5y + 4x = 0$$ **<-Ans**

Homework #2 – Sections 1.4, 1.5, 1.6, 2.1, 2.2 Due 10/12/09

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36. Find the equations (slope-intercept and standard form)of the line
perpendicular to x - y + 2 = 0 and passes through point (3,1).

Get the slope of the line x - y + 2 = 0 by solving for y:
            y = x + 2 => Slope = 1
So the slope of the perpendicular line is -1/1 = -1.
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$$y - y1 = m(x - x1)$$
$$y - 1 = -1(x - 3))$$
$$y = -x + 4 \quad and \quad y + x = 4 \qquad \textbf{<-Ans}$$

9. slope of line : $\frac{\Delta N}{\Delta t} = \frac{5.3 - 5.419}{1993 - 1990} = -\frac{0.119}{3} = -0.0397$

Using pt - slope formula :

$$y - 5.3 = -0.0397\,(x - 1993)$$
$$y = -0.0397x + 1993 \cdot 0.0397 + 5.3$$
$$= -0.0397x + 84.42$$

y = Number of N

x = time

$$\Rightarrow \quad N(t) = -0.0397t + 84.42$$

50. (a) Equation of line from (0,0) to pt of contact $(3, -4)$:

$$y = -\frac{3}{4}x$$

$\Rightarrow$ Slope of line perpendicular to that : $\frac{4}{3}$

$\Rightarrow$ Using pt-slope formula, $y + 4 = \frac{4}{3}(x - 3)$

$$y = \frac{4}{3}x - 4 - 4 = \frac{4}{3}x - 8$$

(b) y-intercept : -8 x-intercept : $x = \frac{8}{\frac{4}{3}} = 6$

(c) length : $\longrightarrow$ $\ell = \sqrt{6^2 + (-8)^2} = \sqrt{36 + 64} = 10$.

Section 4.1

2. $x=3 \quad y=2$

$x=-3 \quad y=-4$

$$m=\frac{-4-2}{-3-3}=1$$

$$y-2=1\times(x-3)$$

$$y=x-1$$

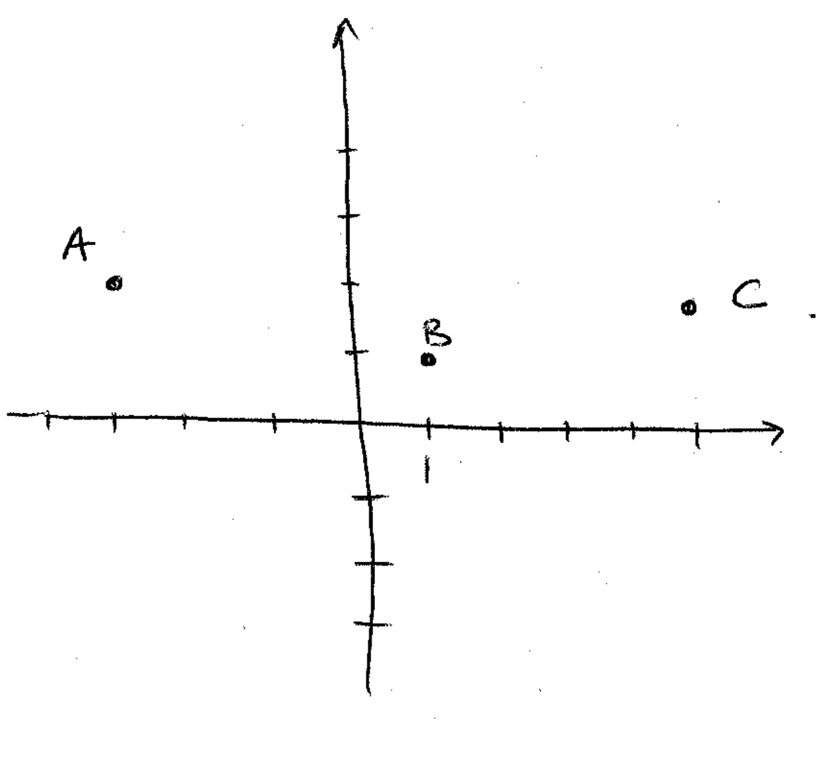
10 A $(-3,2)$, B $(1,1)$ C. $(5,2)$

Slope of line AC: $\frac{2-2}{5-(-3)}=0$

Slope of line BC: $\frac{2-1}{5-1}=\frac{1}{4}$

$\Bigg\}$ slopes are different so pts cannot be on the same line

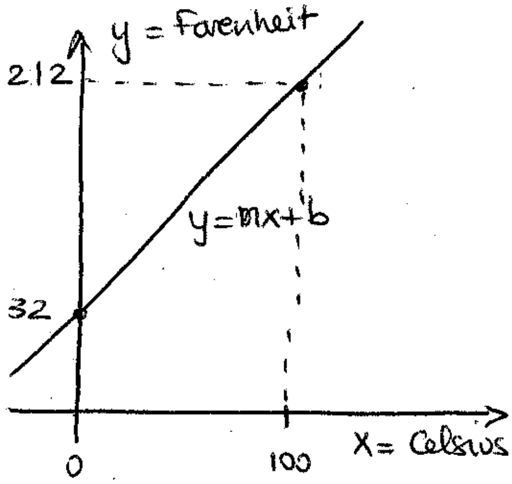
or



$\Rightarrow$ Pts not aligned

[12]

14\. (a) $m=\frac{212-32}{100-0}=\frac{180}{100}=1.8$



[4] $y-32=1.8\times(x-0)$

$y=1.8x+32$

b) $98.6=1.8\times x+32$

[4] $x=\frac{98.6-32}{1.8}$

$x=37°C$

(c) $z=1.8z+32$

[4] $-0.8z=32$

$z=-40°C$

30\. $f(x)=-2810.96x+5,653,063.25$

Firstly, find the line go through the first point and the last point on the table

$m=\frac{53136-69075}{1992-1986}=-2656.5$

$y-69075=-2656.5(x-1986)$

$y=-2656.5x+1986\times2656.5+69075$

$y=-2656.5x+5344884$

the two lines are very close to each other, but they are not exactly the same. This is because the line on the text book is based on all the 7 points, while the line calculated from the first and last two points are based on just two points. The former one is more accurate.

a) $f(x) = -2810.96x + 5,653,063.25$

$x = 1997$

$f(x) = -2810.96 \times 1997 + 5,653,063.25$

$f(x) = 39576.13$

percentage error

$$\frac{39576.13 - 36,110}{36,110} = \frac{3466.13}{36,110} = 9.6\%$$

b) $10,000 = x \cdot (-2810.96) + 5,653,063.25$

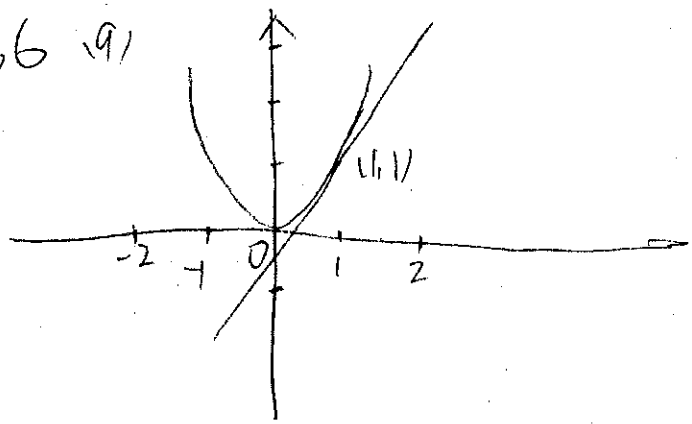
$x = 2007.52$

≈ 2008

$0 = x \cdot (-2810.96) + 5653,063.25$

$x \approx 2011$

36 (a)



(b)

	0.9	0.99	0.999
x	0.9	0.99	0.999
x^2	0.81	0.9801	0.998001
$2x-1$	0.8	0.98	0.998

	1.1	1.01	1.001
x	1.1	1.01	1.001
x^2	1.21	1.0201	1.002001
$2x-1$	1.2	1.02	1.002