Practice Midterm

Name: _____

Calculators are not allowed.

Read all the questions before you start working on any of them. Start with the ones you are most comfortable with, and continue with the other ones later. Always double-check your answers. There is some scrap paper in the back; if you need more, please ask the instructor/proctor.

Relax, and do your best!

PROBLEM 1: SHORT QUESTIONS. In the following questions, you are merely asked to provide the answer. No justification is needed. You should not be spending more than 1 minute per question.

1. What is the equation of the line with slope 4 and $y-$intercept 3?

2. What is the equation of the line perpendicular to $y = 2x - 1$ which goes through the point $(1, 1)$?

Given the functions $f(x) = 2(x - 1)^2$ and $g(x) = \sqrt{3x - 1}$

3. What is $f(a + b)$? _____

4. What is the domain of $f(x)$? _____

5. What is the domain of $g(x)$? _____

6. What is $f \circ g(x)$? _____

7. What is $g \circ f(x)$? _____

8. What is the inverse of $g(x)$? _____

9, 10. Sketch the function $k(x) = (x-2)^3 + 1$ **and** its inverse on the same graph.

11. If $f(x) = x^3 + 2x + 1$, what is $f[f^{-1}(x+1)]$? _____

12. Complete the square for the expression $-x^2+2x+3$: _____

Given the parabola $y = -2(x+2)^2 - 4$:

13. What are the coordinates of the vertex? _____

14. Does it open up or down? _____

15. What is the $y-$intercept? _____

16. What are the $x-$intercepts? _____

17. Based on this information, sketch the parabola $y = 2(x+2)^2 - 4$, making sure to annotate your graph correctly.

18. How many times does this parabola intercept the $x-$axis?

$f(x) = 2x^2 + 6x - 9$? _____

19. Factor the quadratic $2x^2 + 6x - 9$

20. Factor the following expression by grouping, and make sure your result is *fully* factored:
$3x^4 - 6x^3 - x^2 + 2$.

PROBLEM 2: FUNCTIONS AND INVERSES: Consider the function $f(x) = x^2 - 2x$.

(a) What is the name of this kind of function? _____

(b) What are the solutions to $f(x) = 0$? _____

(c) Draw a signs table for $f(x)$

(d) Using this information, sketch $f(x)$.

(e) Explain why, when finding the inverse, we should limit our study to the interval $x \geq 1$.

(f) Solve the equation $x^2 - 2x = y$ for x.

(g) Deduce what $f^{-1}(x)$ is. Hint: the condition $x \geq 1$ should tell you whether to take the $+$ solution or the $-$ solution.

(h) What is the domain of $f^{-1}(x)$? _____

(i) Sketch the functions $f(x)$ (for $x \geq 1$) and $f^{-1}(x)$ (for x in the domain of f^{-1}) on the same graph. Clearly mark which is which.

PROBLEM 3. HIGHER ORDER POLYNOMIAL Consider the higher order polynomial function $x^3 - 4x^2 - 5x$.

(a) What is the behavior near $+\infty$ and $-\infty$?

When x tends to $-\infty$, $f(x)$ goes to _____

When x tends to $+\infty$, $f(x)$ goes to _____

(b) Factor the function: _____

(c) Determine the $x-$ and $y-$ intercepts

$x-$intercept(s): _____ $y-$intercept: _____

(d) Draw a signs table

(e) Sketch the function

PROBLEM 4. APPLIED PROBLEM.

Two friends decide to launch a new social networking website. The initial cost for setting up their company is cheap - a mere $2,000 to buy two laptops on which they develop their software. Once the website is launched, each user which joins the site is asked to click on 2 adverts of their choice, in order to open their account. Each time someone click on 1 advert, the friends make 1 cent (=$0.01).

(a) What is the profit made as a function of the number of users? Call this function $p(n)$.

(b) If 1,000,000 people join, how much do they make?

(c) For the two friends to make $10,000 how many users must join?

(d) For the two friends to make $d dollars, how many users must join ?

(e) What is the inverse function of $p(n)$?