Practice Final

Name: ________________________________

Calculators are not allowed.

Read all the questions before you start working on any of them. Start with the ones you are most comfortable with, and continue with the other ones later. Always double-check your answers.

Relax, and do your best!

PROBLEM 1: SHORT QUESTIONS. [40 POINTS] In the following questions, you are merely asked to provide the answer.

1. What is the equation of the line parallel to the $y = x$ line which goes through the point $(3, 2)$?

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2. What is the distance between the points $(-5, 2)$ and $(1, 1)$

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3. Is this the equation of a circle? $3x^2 - 3y^2 = 2$? YES/NO

4. Does the point $(\sqrt{5/3}, 1)$ lie on the curve defined in question 4? YES/NO.

Given the functions $f(x) = e^x$ and $g(x) = \ln(x - 2)$

5. What is the domain of g? ______________________________________

6. What is $f \circ g(x)$? ______________________________________

7. What is $g \circ f(x)$? ______________________________________

8. Given the function $f(x) = \ln(\sqrt{x})$, what is $f[f^{-1}(x{-}1)]$? ______________________________________

9. Complete the square for the expression $2x^2 - 2x + 1$:

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10.11. Sketch the functions $f(x) = \frac{1}{x-2}$ and $g(x) = \sqrt{x-1} - 1$

Given the parabola $y = x^2 - 2x - 3$:

12. What are the coordinates of the vertex? ____

13. Does it open up or down? ____

14. What is the $y-$intercept? ____

15. What are the $x-$intercepts?

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16. Draw a signs table for the function $f(x) = \frac{(2x-2)(1-x)}{x^2(x+1)}$

17. Given the function $f(x)$ of question 16, what is the domain of $\ln(f(x))$?

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18. Given the function $f(x)$ of question 16, what is the domain of $2^{f(x)}$?

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19. Expand the function $f(x)$ of question 16 into a sum (or difference) of logarithms of the kind $\ln(ax+b)$.

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20. Simplify $f(x) = \frac{2^x 4^{-2x}}{8^{3x} 2^{-x}}$

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21. Simplify $\log_4(2x^2) - 2\log_4(x)$

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22. $\ln(x-y) = \frac{x}{y}$: TRUE /FALSE

23. 24. Sketch the functions $\log_2(x-1)$ and $2^x - 1$.

25. Simplify $\log_2(e^x)$

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26. 27. Sketch the functions x^π and $-x^{-2}$

28. What is the inverse of the function $f(x) = 4x^{-1/4}$?

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29. What is the inverse of the function $f(x) = 2^{x+1} - 3$

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30. What is the inverse of the function $f(x) = \ln(x^2)$

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31. Solve the equation $2^x = 3^{x-1}$

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32. $\log_4(x^3) = \frac{3}{\ln(4)}\ln(x)$: TRUE/FALSE

33. Write $2^{x/a}$ as a natural exponential

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34. Sketch the function $\sin(x)$

35. What is $\tan(\pi/3)$? ____

36. Solve the equation $\sec(x) = 2$.

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37. Simplify the expression $\frac{\sin(2x)}{2\sin(x)} - \frac{1}{\sec(x)}$

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38. Draw all the points on the unit circle where $\cos(x) = -\sin(x)$.

39. What is $\arccos(\cos(x+\pi))$?

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40. What is $\log_{10}(\sin(\pi/2))$?

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PROBLEM 2. INEQUALITIES. [15 POINTS] Solve the inequality

$$\frac{5}{x-2} < 2x+1$$

ANSWER: $x \in$____________________________________

PROBLEM 3: RATIONAL FUNCTIONS.[15 POINTS] Consider the function $f(x) = \frac{1}{x-2} + \frac{1}{(x-3)(x+2)}$.

(a) Reduce to the same denominator, simplify then factor this expression.

ANSWER: __________________________________

(b) What is the domain of $f(x)$? ____

(c) What is the $y-$intercept? ____

(d) What are the $x-$intercepts? ____

(e) Draw a signs table for $f(x)$

(e) What is the behavior of $f(x)$ as x goes to $+\infty$ and $-\infty$?

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(f) Using this information, sketch $f(x)$.

Problem 4. Logarithms and exponentials. [15 points] We consider here the two functions

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

and

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

These functions have many properties which are reminiscent of the trigonometric functions $\cos(x)$ and $\sin(x)$. Here we prove two of them.

(1) Evaluate and simplify the expression $2\cosh(x)\sinh(x)$ to show that it is equal to $\sinh(2x)$.

(2) Evaluate and simplify the expression $\cosh(x)\cosh(y) + \sinh(x)\sinh(y)$ to show that it is equal to $\cosh(x+y)$

PROBLEM 5. TRIGONOMETRIC FUNCTIONS. [15 POINTS]

(a) Use the following addition formula

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$

together with the two double-angle formulas and the Pythagorean formula to prove the identity

$$\cos(3x) = 4\cos^3(x) - 3\cos(x)$$

(b) Solve the equation $4\cos^3(x) - 3\cos(x) = 0$ directly, and write out all the possible solutions.

(c) Solve the equation $\cos(3x) = 0$ directly, and write out all the solutions.

Applied Problem. Note: this is simplified from real tides! Do not rely on your knowledge of Santa Cruz tides to answer the questions! [15 points]

Tides are a regular oscillation of the Ocean's mean water height, a phenomenon rather well-known by Santa Cruz surfers. The mean water height (i.e. the mean water level ignoring waves) varies with time as

$$H(t) = 1.5 \cos\left(\frac{\pi(t-8)}{6}\right)$$

where t is given in hours (t=0 hours being midnight), and H is given in meters.

(a) What is the water height at midnight? ______________________________

(b) What is the difference in water height between high-tide (i.e. when the height is maximum) and low-tide (i.e. when the height is minimum)?

(c) What is the period of the tide?

(d) At what times of the day is high-tide?

(e) Based on this, sketch the function $H(t)$ for t between 0 and 24.