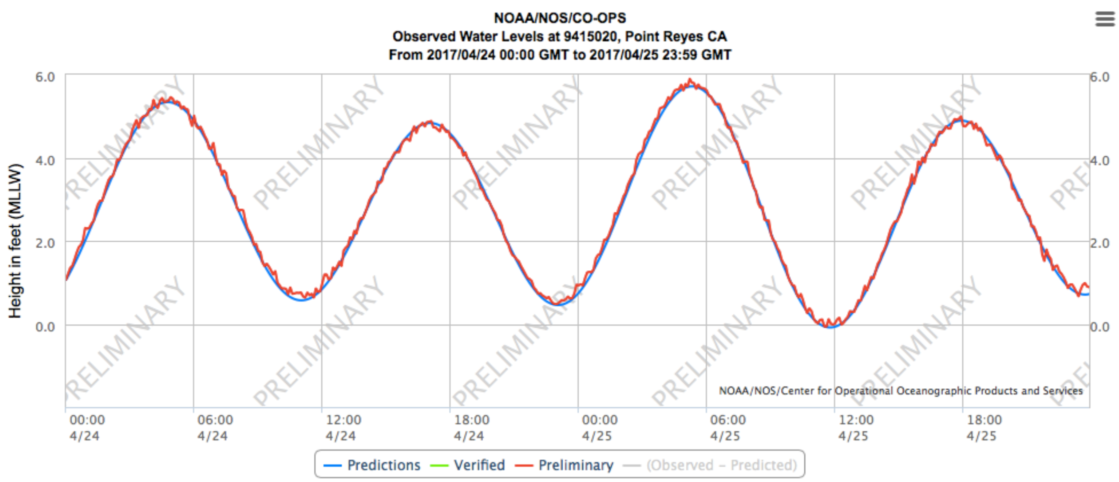
6.4 Periodic functions

Textbook Section 5.6

6.4.1 Case Study: *Tides*

The National Oceanic and Atmospheric Administration studies the tidally-induced variation of the water level with time in various coastal areas around the US, including Point Reyes in CA. They provide both forecasting services (i.e. predictions of the future water level) and monitoring services (i.e. measuring the actual water level). The following figure shows the result of one of their predictions and monitoring efforts, for the 48-hour period starting at midnight on April 24th, 2017.



We see that

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While it is actually quite difficult to model this phenomenon as accurately as NOAA did, we can try to model it approximately using sine or cosine functions. Let's relabel the hours starting at time $t = 0$, and increasing monotonically up to 48. Once that is done, how can we create a function that approximately models the data?

On the other hand here we have a function which is has the properties

To model the tides function starting from the sine function, we therefore have to:

Using simple transformations on sine or cosine functions, we can therefore model many oscillatory functions. Let us now this how to do this in somewhat more generality.

6.4.2 Oscillatory functions

We now generalize what we saw in the previous section: the functions

are oscillatory functions where:

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These constants cause a vertical shift, a horizontal shift, and a vertical stretch of the function respectively. The interpretation of the constant b is a little less obvious, although we saw that it causes a horizontal stretching of the function. In fact, b is called the *frequency* of the oscillation, and it is related to the length of the pattern of oscillation, called the *period* of the oscillation.

DEFINITION:

To understand the relationship between the period of an oscillatory function p and the frequency b, note that

EXAMPLE: What are the mean, amplitude, period, frequency and shift of the following functions:

- $f(x) = 2 + 2\cos(2x + 2)$
- $f(x) = 3 + e\sin(\pi x)$
- $f(t) = \sin(2\pi t - 1)$
- $h(t) = 2\sin\left[\frac{2\pi}{12}(x - 2)\right] + 3$

To summarize, here are the diagrams corresponding to the functions $f(x) = m + a\sin(b(x - c))$ and $g(x) = m + a\cos(b(x - c))$.

6.4.3 Periodic functions

The definition of a period can be extended to any function that has a pattern that repeats itself over a certain period, even if that function does not arise from a sine or a cosine function.

DEFINITION:

EXAMPLE

- The function $\tan(x)$ is periodic with period π. Indeed,

- Here are some real-life examples of periodic functions that are more complex than simple sinusoidal functions: