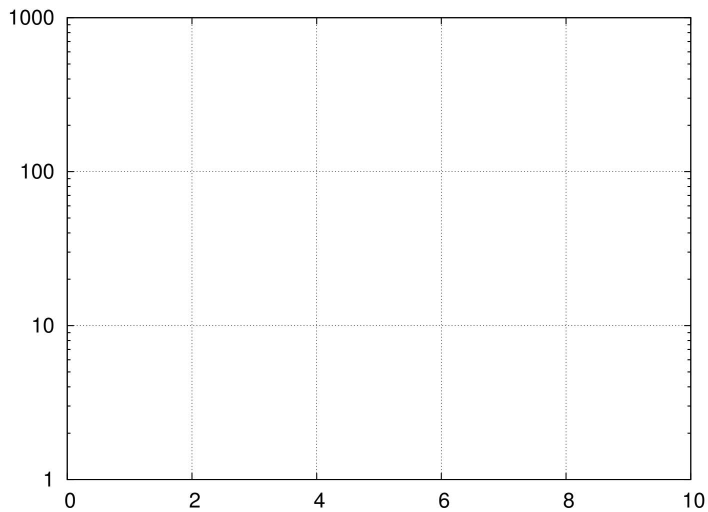
5.4 Logarithmic axes

In the various case studies we have seen in the last 2 weeks, we came across logarithmic axes and their peculiar properties when it comes to graphing power laws and exponential functions. Let's see this once again through two examples.

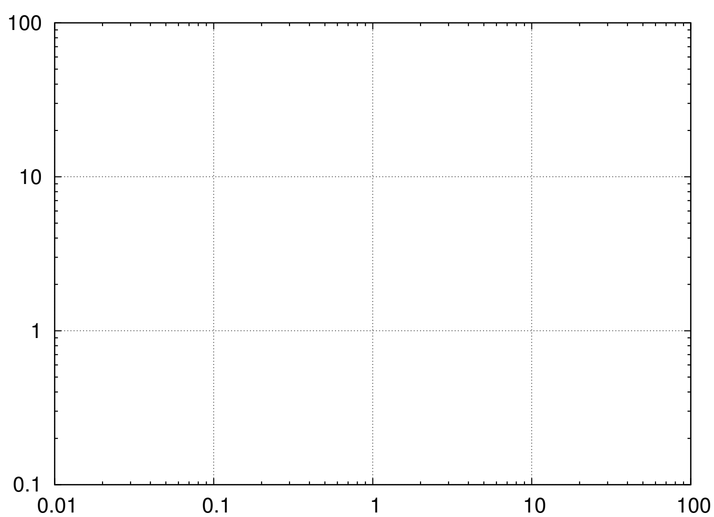
Example 1: Graphing an exponential function on log-linear axes.

Let's draw the function $N(x) = 2 \times 4^x$ (the rabbit population growth model) on a graph with a logarithmic y-axis, and a linear $x-$axis (called a Log-Linear graph).



Example 2: Graphing a power law on log-log axes.

Let's draw the function $N(x) = \frac{2}{x}$ on a graph with a logarithmic $x-$ and y-axes, (called a Log-Log graph).



These findings confirm what we found in earlier lectures, namely that

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This is in fact generally true, and it is relatively easy to understand why this is the case.

Let us first look at the case of exponential functions.

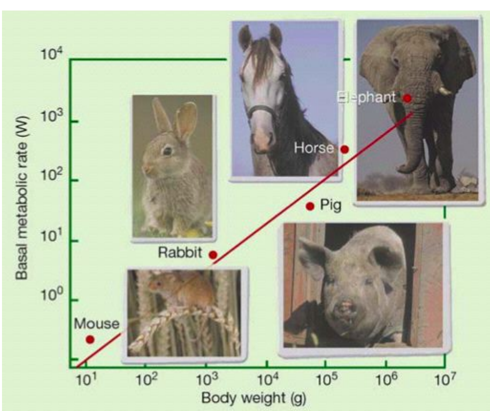
Similarly, for power law functions, we have:

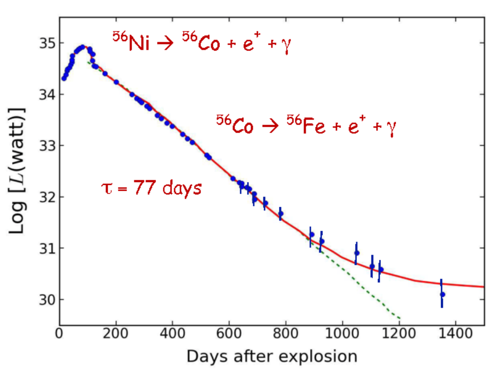
Knowing that a logarithmic axis actually records the logarithm of a quantity (rather than the quantity itself) also explains why the numbers 0.1, 1, 10, 100, 1000 etc are equidistant on that axis. Indeed,

To summarize, we have the following results:

- The logarithmic axis actually plots the logarithm of a quantity $\log x$ (or $\log y$), though often labels it as x (or y).
- The graph of an exponential function $f(x) = be^{\pm rx}$ appears as a straight line on a log-linear plot. The slope of the line is equal to $\pm r$.
- The graph of a power function $f(x) = bx^a$ appears as a straight line on a log-log plot. The slope of the line is the exponent a, and is positive if $a > 0$, and negative if $a < 0$.

Graphing on log-linear or log-log plots is used in a vast range of scientific papers. to *establish* that a given dataset is best modeled by a power law or an exponential. For instance:





5.5 The Gaussian function

One of the most important functions in many applied fields is the Gaussian function, also called Normal function or more commonly called the bell curve. The function is given by the following formula:

We will now see a Case Study that uses this function.

5.5.1 *Case Study: The distribution of IQ test scores*

IQ tests are standardized tests designed in the late 19th and early 20th century to assess a person's "intelligence". Modern IQ tests measure several factors which are related to intelligence, namely logical reasoning, math skills, language abilities, spatial relations skills, knowledge retained and the ability to solve novel problems.

The result of these tests is a single numeric score x. A distribution of scores can be created by recording how many people scored a particular value x, and dividing that value by the total number of test takers. This creates a function $N(x)$, which records the percentage of people whose score on the test was exactly x. The tests have been designed so that the distribution of test results (given a large enough sample of test-takers), is

$$N(x) = \frac{1}{15\sqrt{2\pi}} e^{-\frac{(x-100)^2}{2(15)^2}}$$

Based on the definition above, this is a Gaussian function with

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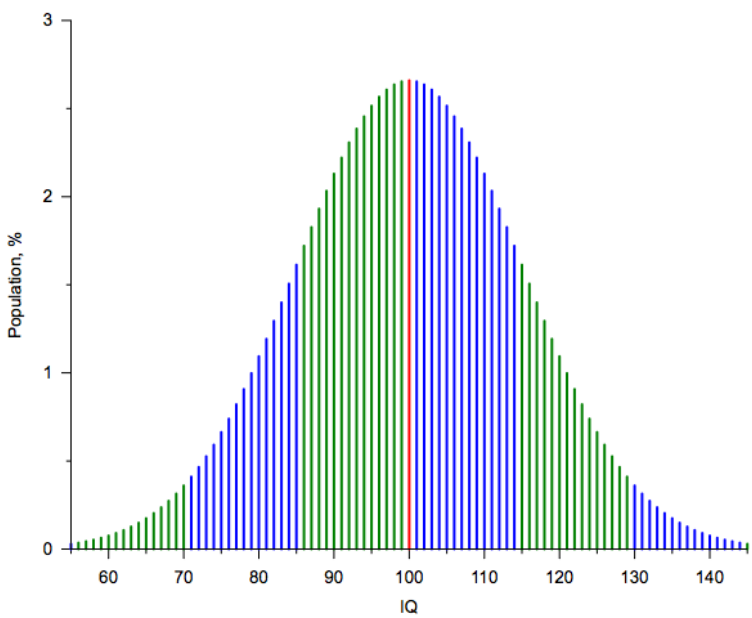
Using this function, we can now answer a few questions. For instance, what is the percentage of people who scored exactly 100 points (the most likely score)?

To get into MENSA, your score on the IQ test must be above 130. What is the percentage of people whose score is exactly 130?

What scores can you get with a probability of exactly 1 *percent?*

We can now check our answers on the graph of the function $N(x)$*, which has the well-known Bell-Curve shape:*



We therefore see that while it is likely that someone will score a value close to 100, scores much lower than, say 70, or greater than, say, 130, are fairly unlikely. This is a defining standard property of the Gaussian function: it is a function that peaks at a certain value, and drops to 0 quite rapidly on either side of that value. Let us now see other properties of the Gaussian.

5.5.2 General properties of the Gaussian function

The Gaussian function has the following properties:

THE MAXIMUM OF THE GAUSSIAN.

THE WIDTH OF THE GAUSSIAN.

Based on this we see that different Gaussians can be obtained by changing the amplitude, mean and width of the Gaussian.