2.1.5 Case study: Cost and revenue: how to decide whether a market is viable

Donnelly's in Santa Cruz sell some of the finest chocolates in the US. The chocolates are sold at $3 a piece. The rental of the facility costs $200 per day, the salaries & benefits of the owners and workers costs $300/day and the raw material for each chocolate costs $1 per piece. How many chocolates do they have to sell per day to break even? What will be the amount of money involved (in and out) at that point?

2.1.6 Mathematical corner: Solutions of linear equations

In many real life applications, we may be led to solve a system of linear equations, i.e. two linear equations for two unknowns. We just saw one example:

However, more generally, we may have to solve a set of equations of the kind

To do so, simply note that y is the same in both equations, so we can set them equal to one another and solve for x. We effectively eliminate one of the unknowns by doing that, and just solve for the other. We get

It's interesting to note that if $a - c = 0$, then there is a problem! To see why, let's interpret this system of equations geometrically.

We see that

-
-

Once the solution for x is known (assuming it exists and it is unique) we can use it to find y. Note that we can use either of the two original equations to do so, and get the same result:

Even more generally, we may have to solve coupled systems of the kind

Note that this last set can be reduced to the one before by solving each equation for y first, and then following the same steps as before.

Examples:

- Solve the system of equations $y = 3x + 1$, and $y = -2x - 4$

- Solve the system of equations $y = x + 1$, and $y = x + 4$

- Solve the system of equations $2x + 3y = 1$, and $x - 3y = 2$

- Solve the system of equations $2x + 4y = 1$, and $x + 2y = \frac{1}{2}$