AMS 3: Practice Midterm

Name: _____

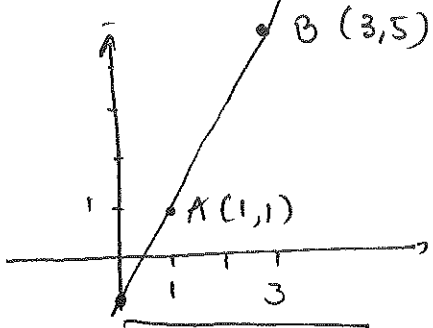
Calculators are not allowed.

Quickly read all the questions before you start working on any of them. Start with the ones you are most comfortable with, and continue with the other ones later. Always double-check your answers.

Relax, and do your best!

PROBLEM 1: SHORT QUESTIONS [30 POINTS] In the following questions, you are merely asked to provide the answer. No justification is needed. You should not be spending more than a 2 minutes per question. Each question is worth 3 points.

1. Find the linear function $f(x)$ such that $f(1) = 1$ and $f(3) = 5$.



Slope $m = \frac{y_B - y_A}{x_B - x_A} = \frac{5-1}{3-1} = \frac{4}{2} = 2$

Point-slope formula:

$y - y_A = m(x - x_A) \Rightarrow y - 1 = 2(x-1)$

$y = 1 + 2x - 2 = 2x - 1 = \boxed{f(x) = 2x - 1}$

2. 3. For which value(s) of the parameter a does this function have exactly one root? $f(x) = x^2 + ax + 1$?

$a =$ ± 2

need $D = a^2 - 4(1)(1) = a^2 - 4 = 0$

$\Rightarrow a = \pm 2$

Given the functions $f(x) = \frac{1}{x^2-1}$ and $g(x) = \frac{1}{x-3}$

4., 5. Write down, combine, and then factor the expression $g(x^2) - f(x)$.

$$g(x^2) - f(x) = \frac{1}{x^2-3} - \frac{1}{x^2-1} = \frac{x^2-1-x^2+3}{(x^2-3)(x^2-1)}$$

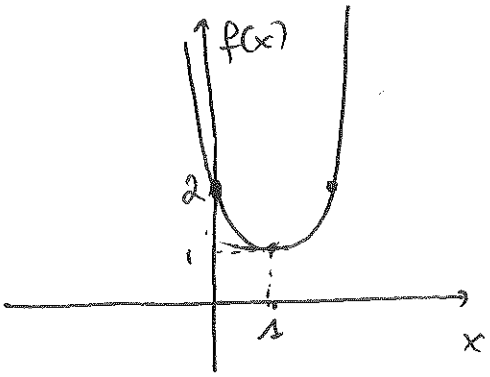
$$= \frac{2}{(x-\sqrt{3})(x+\sqrt{3})(x-1)(x+1)}$$

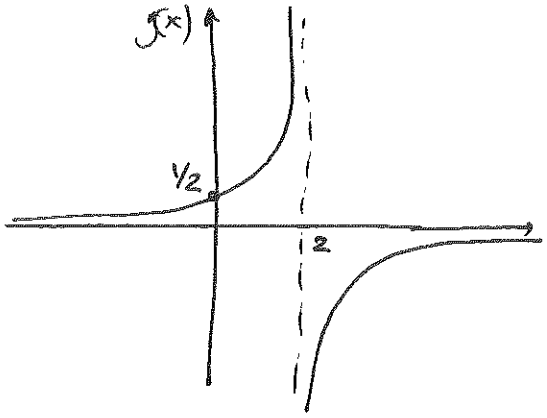
6. What is the domain of $f(x)$? $x \neq -1$ and $x \neq 1$

all answers ok

- $(-\infty, -1) \cup (-1, 1) \cup (1, +\infty)$
- $\mathbb{R} - \{-1, 1\}$

7.,8. Sketch the functions $f(x) = (x-1)^4 + 1$ and $g(x) = \frac{1}{2-x}$





Given the function $f(x) = -2x^2 + 3x + 1$ and its graph:

9. What is the $x-$coordinates of the vertex? $-\frac{b}{2a} = \frac{-3}{2(-2)} = \frac{3}{4}$

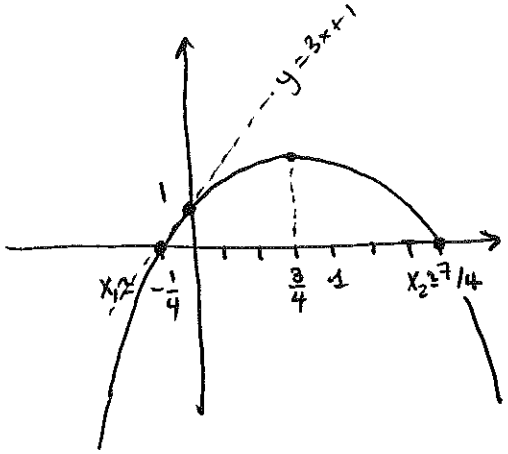
10. Does the parabola open up or down? down

11. What is the equation of the tangent at the $y-$intercept? $y = 3x + 1$

12., 13. Does the function have roots? If so, what are they? $D = 3^2 - 4(-2)(1) = 9 + 8 = 17$

$$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{17}}{2(-2)} = \frac{-3 \pm \sqrt{17}}{-4} = \frac{3 \mp \sqrt{17}}{4}$$

14., 15. Based on this information, sketch the parabola $y = -2x^2 + 3x + 1$, making sure to annotate your graph correctly. Hint: $\sqrt{17}$ is pretty close to 4.



$$\frac{3-\sqrt{17}}{4} \simeq -\frac{1}{4}$$

$$\frac{3+\sqrt{17}}{4} \simeq \frac{7}{4}$$

PROBLEM 2. [20 POINTS] Consider the function $f(x) = 2x^3 - 3x^2 - 2x$.

(a) Behavior for large x:

- What is $f(x)$ approximately equal to for large $|x|$? $f(x) \approx 2x^3$
- What is the equation of the tangent line to $f(x)$ at $x = 0$? $y = -2x$

(b) Is the function odd, even or neither? neither

(c) Factor $f(x)$ $\quad f(x) = x(2x^2 - 3x - 2)$

$$D = (-3)^2 - 4(2)(-2) = 9 + 16 = 25$$

$$x_{1,2} = \frac{-(-3) \pm \sqrt{25}}{2(2)} = \frac{3 \pm 5}{4} = \begin{cases} 2 \\ -1/2 \end{cases}$$

$$\rightarrow 2x^2 - 3x - 2 = 2(x-2)(x+\tfrac{1}{2})$$

$$f(x) = 2x(x-2)(x+\tfrac{1}{2})$$

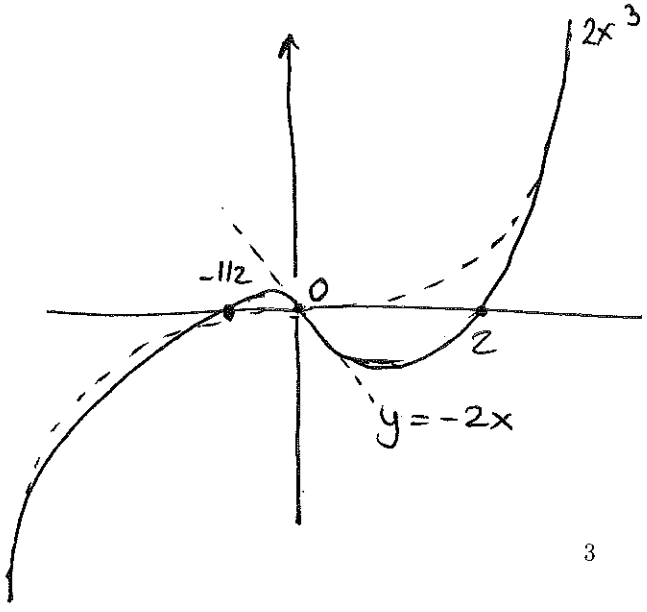
(d) Determine the $x-$ and $y-$ intercepts

$x-$intercept(s): $0, 2, -1/2$ $y-$intercept: 0

(e) Draw a signs table (make sure to include the zeros)

	$-1/2$		0		2	
$2x$	$-$	$-$	0	$+$		$+$
$x-2$	$-$	$-$		$-$	0	$+$
$x+\frac{1}{2}$	$-$	0	$+$	$+$		$+$
	$-$	0	$+$	0	$-$	0 $+$

(f) Sketch the function $f(x)$, making sure to annotate all the important points.



PROBLEM 3. [20 POINTS] Consider the function $f(x) = \frac{x^2-4x}{x^2-9}$.

(a) What is $f(x)$ approximately equal to for large $|x|$? $\frac{x^2}{x^2} = 1$

(b) Factor $f(x)$

$$f(x) = \frac{x(x-4)}{(x+3)(x-3)}$$

(c) Determine the $x-$ and $y-$ intercepts

$x-$intercept(s): $0, 4$ $y-$intercept: 0

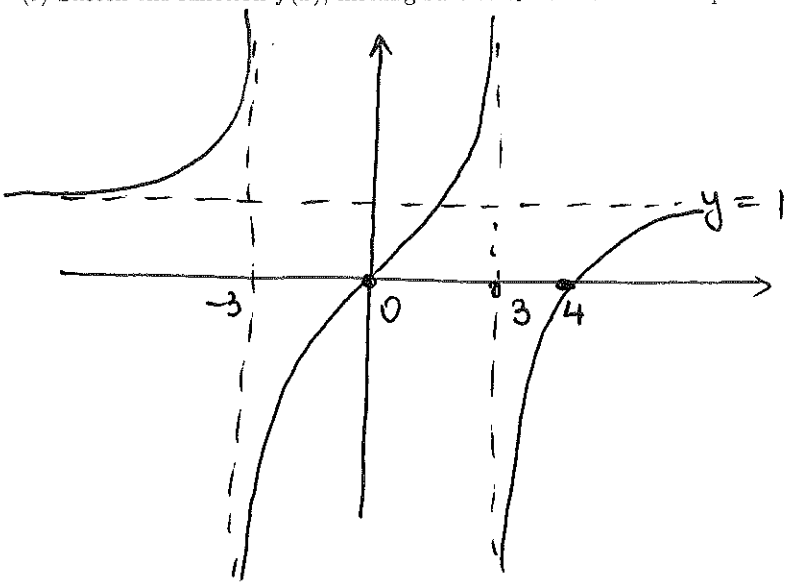
(d) What are the vertical asymptotes?

Vertical asymptotes at $x =$: $-3, 3$

(e) Draw a signs table (make sure to include the zeros and the asymptotes)

	$-\infty$		-3		0		3		4		$+\infty$
x	$-$		$-$		0	$+$	$+$		$+$		$+$
$x-4$	$-$		$-$		$-$		$-$		0	$+$	$+$
$x+3$	$-$	0	$+$		$+$		$+$		$+$		$+$
$x-3$	$-$		$-$		$-$		0	$+$	$+$		$+$
	$+$	∞	$-$		0	$+$	∞	$-$	0	$+$	

(f) Sketch the function $f(x)$, making sure to annotate all the important points.



PROBLEM 4. APPLIED PROBLEM [30 POINTS].

Medical and psychological studies have measured the effect of caffeine on students taking tests, and have found that while a small amount of caffeine can be beneficial and increases alertness, too much caffeine is seriously detrimental and causes a decrease in test performance. They have concluded that the percentage increase in a student's test score p can be modeled as the following function of the caffeine intake x:

$$p(x) = -5x^2 + 20x \tag{1}$$

where x is the number of cups of coffee (standard 12 ounce cup) taken within 3 hours of the test. Note that a negative value of $p(x)$ simply represents a detrimental effect on test score.

Question 1: What kind of function is $p(x)$? quadratic

Question 2: What is the percentage increase in test score after 1 cup of coffee? $p(1) = -5 + 20 = 15$ (percent)

Question 3: What is the optimal number of cups to drink before the test?

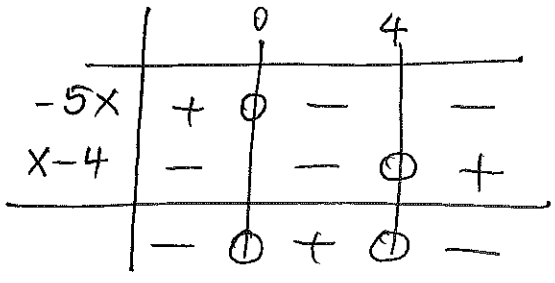
x_v = vertex position $= -\frac{b}{2a} = -\frac{20}{2(-5)} = \frac{-20}{-10} = 2$

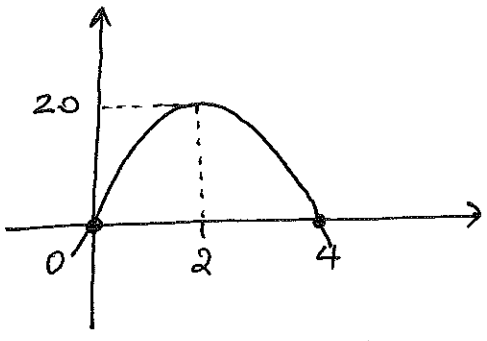
Question 4: After drinking the optimal number of cups, what percentage increase in score can students expect ?

$p(2) = -5(2^2) + 20(2) = -20 + 40 = 20$ (percent)

Question 5: Factor the function $p(x)$. $-5x(x-4)$

Question 6: Sketch the function $p(x)$, making sure to mark the roots and the vertex.





Question 7: After how many cups does drinking coffee become detrimental to performance? 4