AMS 3: Midterm Fall 2017

Name: _____

Calculators are not allowed.

Quickly read all the questions before you start working on any of them. Start with the ones you are most comfortable with, and continue with the other ones later. WORK MUST BE SHOWN FOR ALL ANSWERS. Always double-check your answers.

Relax, and do your best!

PROBLEM 1: SHORT QUESTIONS [30 POINTS] Each question is worth 2 points.

1. Find the linear function $f(x)$ such that $f(5) = \frac{1}{2}$ and $f(2) = -1$.

$A(5, \frac{1}{2})$ $B(2, -1)$

[1] $m = \frac{y_B - y_A}{x_B - x_A} = \frac{-1 - \frac{1}{2}}{2-5} = \frac{-\frac{3}{2}}{-3} = \frac{1}{2}$

line: $y - y_A = m(x - x_A) \Rightarrow y = y_A + m(x - x_A) = \frac{1}{2} + \frac{1}{2}(x-5)$

$= \frac{1}{2} + \frac{1}{2}x - \frac{5}{2} = \frac{1}{2}x - 2$

[1] So $\boxed{f(x) = \frac{x}{2} - 2}$

2.. Under which condition does the following quadratic equation $dx^2 + bx - a = 0$ have two solutions?

[1]

[2] need $D = b^2 - 4(d)(-a) = b^2 + 4ad > 0$

[2] NP 3. Is the function $f(x) = x^2 - 2x + 1$ odd, even, or neither ? neither

4. Simplify $f(x) = \frac{\frac{x+1}{x^2-1}}{x-1} = \frac{\frac{x+1}{(x+1)(x-1)}}{x-1} = \frac{1}{(x-1)^2}$

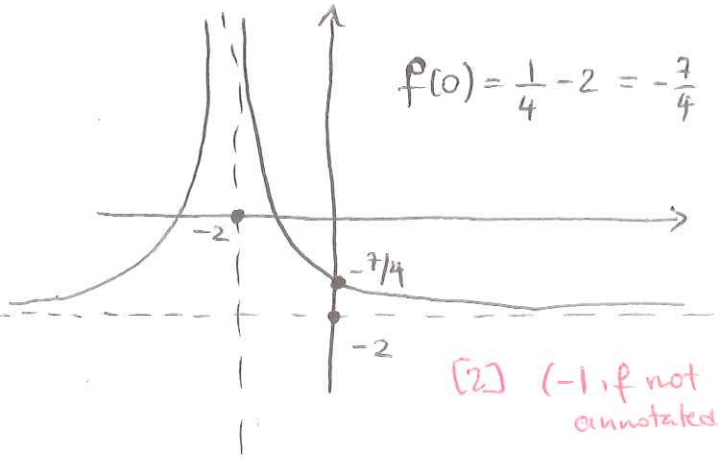
[1] [1]

[2] 5. What is the domain of $\sqrt{\frac{1}{x-1}}$? $(1, +\infty)$

need $x - 1 > 0$ so $x > 1$

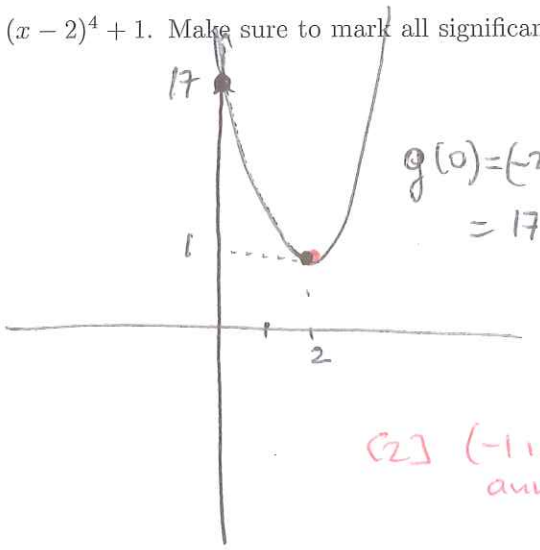
(if $\geq$ or wrong bracket, -1)

6.,7. Sketch the functions $f(x) = \frac{1}{(x+2)^2} - 2$ and $g(x) = (x-2)^4 + 1$. Make sure to mark all significant points and asymptotes on the graph.



$f(0) = \frac{1}{4} - 2 = -\frac{7}{4}$

[2] (-1 if not annotated)



$g(0) = (-2)^4 + 1 = 17$

[2] (-1 if not annotated)

Given the function $f(x) = 2x^2 - 3x - 3$ and its graph:

8. What is the $x-$coordinates of the vertex? 3/4

$x_V = -\frac{b}{2a}$ [1] $= \frac{-(-3)}{2(2)} = \frac{3}{4}$ [1]

[2] NP 9. Does the parabola open up or down? up

[2] NP 10. What is the equation of the tangent at the $y-$intercept? $y = -3x - 3$

11., 12. Does the function have roots? If so, what are they?

[2] $D = b^2 - 4ac = (-3)^2 - 4(2)(-3) = 9 + 24 = 33$

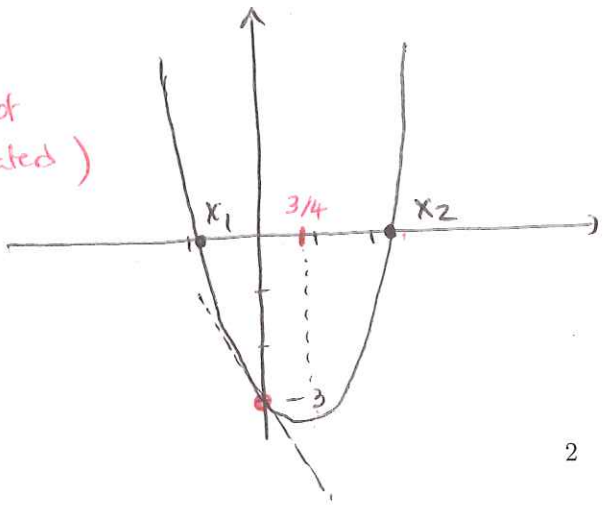
[2] $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-3) \pm \sqrt{33}}{2(2)} = \frac{3 \pm \sqrt{33}}{4}$

13. Factor the function $f(x) = 2x^2 - 3x - 3$: $2\left(x - \frac{3-\sqrt{33}}{4}\right)\left(x - \frac{3+\sqrt{33}}{4}\right)$

[2] (-1 for errors)

14., 15. Based on this information, sketch the parabola $y = 2x^2 - 3x - 3$, making sure to annotate your graph correctly. Hint: $\sqrt{33}$ is pretty close to 6.

[4] (-1 if not annotated)



$x_2 = \frac{3+\sqrt{33}}{4} \approx \frac{3+6}{4} \approx \frac{9}{4} \approx 2.25$

$x_1 = \frac{3-\sqrt{33}}{4} \approx \frac{3-6}{4} \approx -\frac{3}{4} \approx -0.75$

PROBLEM 2. [20 POINTS] Consider the function $f(x) = (1-x)(x^2+2x+1)$.

(a) Expand $f(x)$ [4]

$f(x) = x^2 + 2x + 1 - x^3 - 2x^2 - x$

$= -x^3 - x^2 + x + 1$

(-1 per error)

(b) Behavior for large x.

[1] • What is $f(x)$ approximately equal to for large $|x|$? $-x^3$

[1] • What is the equation of the tangent line to $f(x)$ at $x = 0$? $y = x + 1$

(c) Finish factoring $f(x)$

[3] $f(x) = (1-x)(x+1)^2$

(d) Determine the $x-$ and $y-$ intercepts

[1][1][1] $x-$intercept(s): 1 and -1 $y-$intercept: 1

(e) Draw a signs table (make sure to include the zeros)

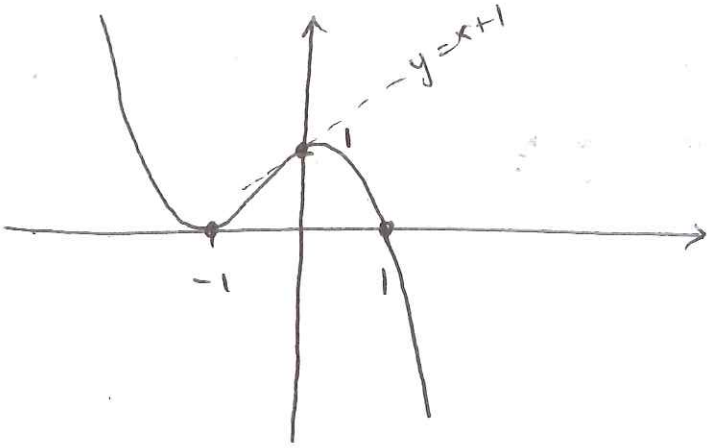
[4]

		-1		1	
$(1-x)$	+		+	0	−
$(1+x)^2$	+	0	+		+
f	+	0	+	0	−

(-1 per wrong line or column)

(f) Sketch the function $f(x)$, making sure to annotate all the important points.

[4]



(-1 per missing annotation)

PROBLEM 3. [20 POINTS] Consider the function $f(x) = \frac{x^2-2x}{x^2-9}$.

[2] (a) What is $f(x)$ approximately equal to for large $|x|$? 1

(b) Factor $f(x)$

[4] $f(x) = \dfrac{x(x-2)}{(x-3)(x+3)}$

(c) Determine the $x-$ and $y-$ intercepts

[0][0][0] $x-$intercept(s): 0, 2 $y-$intercept: 0

(d) What are the vertical asymptotes?

[0][1] Vertical asymptotes at $x =$: 3, -3

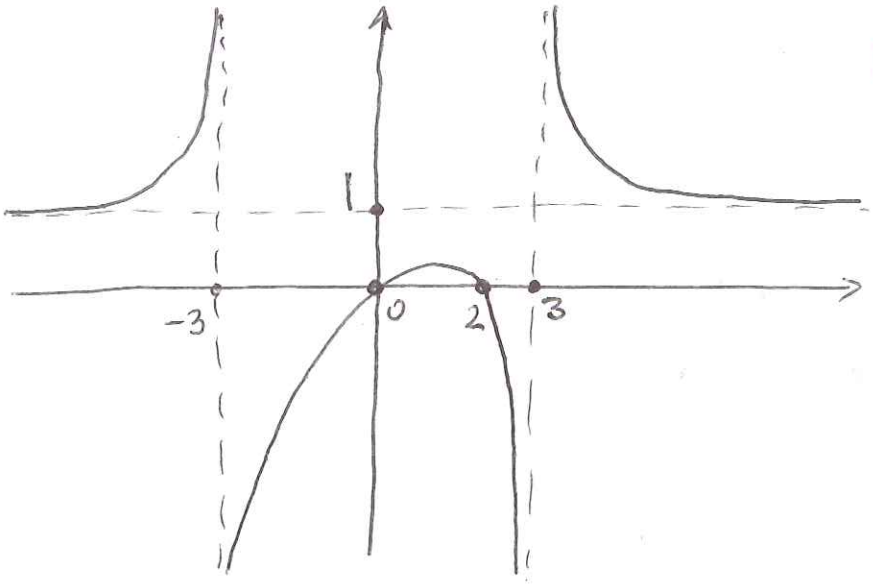
(e) Draw a signs table (make sure to include the zeros and the asymptotes)

[4]

		−3		0		2		3	
x	−		−	0	+	+	+		
$x-2$	−		−		−	0	+	+	
$x-3$	−		−		−		−	0	+
$x+3$	−	0	+		+		+		+
	+	0	−	0	+	0	−	0	+

(f) Sketch the function $f(x)$, making sure to annotate all the important points.

[5]



(-1 per missing annotation)

PROBLEM 4. APPLIED PROBLEM [30 POINTS].

A market analysis has shown that the daily revenue R of Donelli's chocolate company depends on the price p they sell their chocolates as

$$R(p) = -100p^2 + 500p - 400$$

[2] **Question 1:** What type of function is $R(p)$? quadratic

Question 2: How much money would they make each day if they sold their chocolates 2 dollars a piece?

[5]
$$R(2) = -100(2)^2 + 500(2) - 400$$
$$= -400 + 1000 - 400 = 200$$
$$\rightarrow \$200/\text{day}$$

Question 3: What price p should they pick to maximize their daily revenue?

[5] → look for x-coordinate of vertex,

$$p_{max} = -\frac{b}{2a} = \frac{-500}{2(-100)} = \frac{-500}{-200} = 2.5$$

Question 4: What are the lowest and highest possible values of p they can use and still have a positive revenue?

[10] → for which values of $R(p)$ is $R(p)$ positive? → factor $R(p)$

$$R(p) = -100(p^2 - 5p + 4) = -100(p-1)(p-4)$$

		1		4	
-100	$-$		$-$		$-$
$p-1$	$-$	0	$+$		$+$
$p-4$	$-$		$-$	0	$+$
R	$-$	0	$+$	0	$-$

→ positive for $\boxed{1 < p < 4}$

Question 5: Using this information, graph the function $R(p)$.

[8]