AMS 3: Midterm 2017

Name: _____

Calculators are not allowed.

Quickly read all the questions before you start working on any of them. Start with the ones you are most comfortable with, and continue with the other ones later. Always double-check your answers.

Relax, and do your best!

PROBLEM 1: SHORT QUESTIONS [30 POINTS] In the following questions, you are merely asked to provide the answer. No justification is needed. You should not be spending more than a 2 minutes per question. Each question is worth 2 points.

[2] 1. Find the linear function $f(x)$ such that $f(2) = 1$ and $f(1) = -3$.

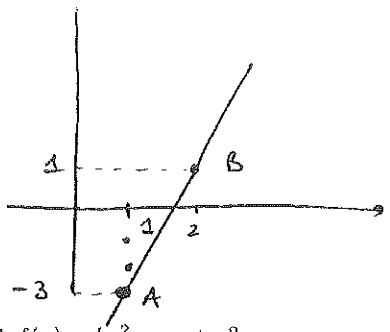
slope $m = \frac{y_B - y_A}{x_B - x_A} = \frac{1-(-3)}{2-1} = \frac{4}{1} = 4$ [1]

point-slope formula

$y - y_A = m(x - x_A)$

$y - 1 = 4(x-2)$

$y = 4x - 8 + 1 = 4x - 7$ [1]



2.. Under which condition does the following quadratic have only one root? $f(x) = bx^2 - cx + a$?

[2] N.P. $D = (-c)^2 - 4(b)(a) = 0 \Rightarrow c^2 - 4ab = 0$

Given the functions $f(x) = \frac{1}{x+1}$ and $g(x) = \frac{1}{x-3}$

[4] 3., 4. Write down, combine, and then factor the expression $g(x) - f(x)$.

$\frac{1}{x-3} - \frac{1}{x+1} = \frac{x+1-(x-3)}{(x-3)(x+1)} = \frac{x+1-x+3}{(x-3)(x+1)} = \frac{4}{(x-3)(x+1)}$

[2] [2]

5. What is the domain of $\sqrt{f(x)}$? $x > -1$ or $(-1, +\infty)$

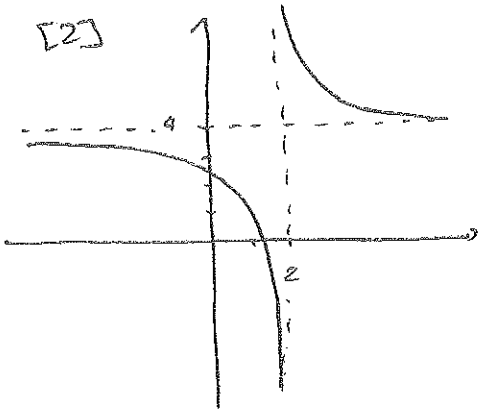
[2]

Needed $x+1 > 0$ so $x > -1$

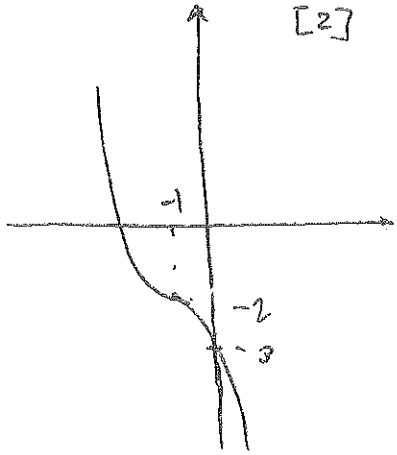
-1 for forgetting to exclude =1

[4] 6.,7. Sketch the functions $f(x) = \frac{1}{x-2} + 4$ and $g(x) = -(x+1)^3 - 2$

[2]



[2]



Given the function $f(x) = 2x^2 - 2x - 1$ and its graph:

[2] 8. What is the x–coordinates of the vertex? 1/2

N.P. $-\frac{b}{2a} = \frac{-(-2)}{2(2)} = \frac{2}{4} = \frac{1}{2}$

[2] N.P. 9. Does the parabola open up or down? up

[2] 10. What is the equation of the tangent at the y–intercept? $y = -2x - 1$

[4] 11., 12. Does the function have roots? If so, what are they?

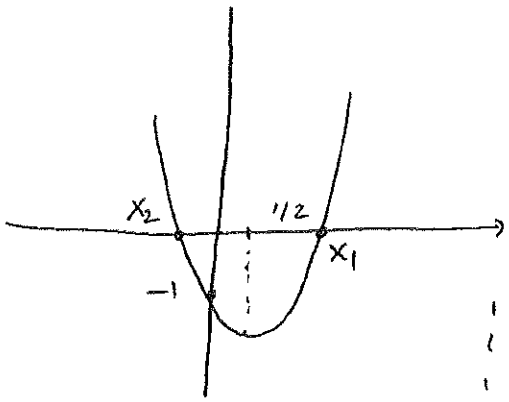
$D = (-2)^2 - 4(2)(-1) = 4 + 8 = 12$ [2]

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-2) \pm \sqrt{12}}{2(2)} = \frac{2 \pm \sqrt{12}}{4}$ [2]

[2] 13. Factor the function $f(x) = 2x^2 - 2x - 1$: $2(x - x_1)(x - x_2) = 2\left(x - \frac{2+\sqrt{12}}{4}\right)\left(x - \frac{2-\sqrt{12}}{4}\right)$

-1 if forget "2"

[4] 14., 15. Based on this information, sketch the parabola $y = 2x^2 - 2x - 1$, making sure to annotate your graph correctly. Hint: $\sqrt{12}$ is pretty close to 3.5.



$x_1 \simeq \frac{2+3.5}{4} \simeq \frac{5.5}{4}$

$x_2 \simeq \frac{2-3.5}{4} \simeq \frac{-1.5}{4}$

1 for vertex
1 for y intercept
1 for x_1, x_2
1 for shape of graph

PROBLEM 2. [20 POINTS] Consider the function $f(x) = (1-x)(x^2-4)$.

[2] (a) Expand $f(x)$

$$f(x) = x^2 - 4 - x^3 + 4x \quad = \quad -x^3 + x^2 + 4x - 4$$

(b) Behavior for large x.

[1] • What is $f(x)$ approximately equal to for large $|x|$? $-x^3$

[1] • What is the equation of the tangent line to $f(x)$ at $x = 0$? $y = 4x - 4$

[4] (c) Finish factoring $f(x)$

$$f(x) = -(x-1)(x-2)(x+2)$$

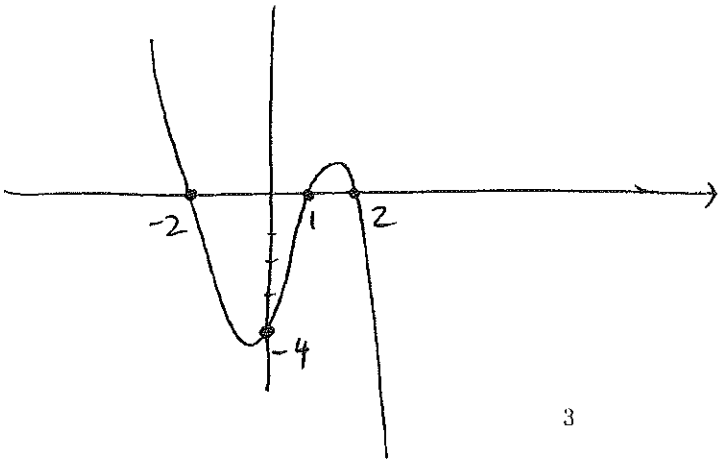
[4] (d) Determine the $x-$ and $y-$ intercepts

$x-$intercept(s): $1, 2, -2$ $y-$intercept: -4

[4] (e) Draw a signs table (make sure to include the zeros)

	-2		1		2	
-1	$-$		$-$		$-$	$-$
$(x-1)$	$-$		$-$	0 $+$		$+$
$(x-2)$	$-$		$-$		$-$ 0	$+$
$(x+2)$	$-$ 0	$+$		$+$		$+$
	$+$ 0	$-$	0	$+$	0	$-$

[4] (f) Sketch the function $f(x)$, making sure to annotate all the important points.



PROBLEM 3. [20 POINTS] Consider the function $f(x) = \frac{x-1}{3x-x^2}$

[2] (a) What is $f(x)$ approximately equal to for large $|x|$? $\frac{1}{x}$

[4] (b) Factor $f(x)$

$$f(x) = \frac{x-1}{x(3-x)}$$

(c) Determine the $x-$ and $y-$ intercepts

[2] $x-$intercept(s): 1 $y-$intercept: none

(d) What are the vertical asymptotes?

[2] Vertical asymptotes at $x =$: 0, 3

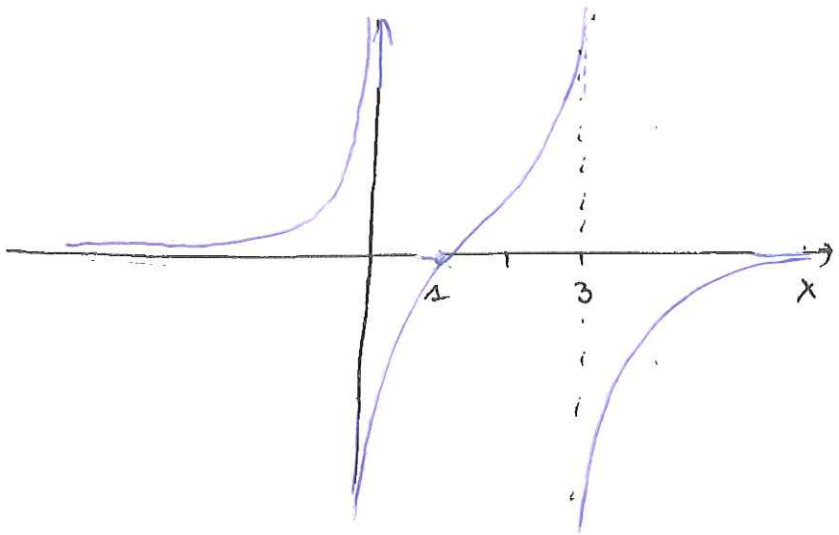
(e) Draw a signs table (make sure to include the zeros and the asymptotes)

[4]

	0	1	3	
$x-1$	−	0 −	0 +	+
x	0 −	+	+	+
$3-x$	+	+	0 +	0 −
	+ ∄	0 −	0 +	∄ −

(f) Sketch the function $f(x)$, making sure to annotate all the important points.

[6]



PROBLEM 4. APPLIED PROBLEM [30 POINTS].

This problem guides you through the calculation of the optimal sales price for a new HP tablet. Market analysis shows that the likely number of tablet N that would sell in the US every day is a linear function of the sales price x:

$$N(x) = -2x + 4000$$

[2] **Question 1:** How many tablets would be sold (per day) if the price was \$500 per tablet ?

$N(500) = -2 \cdot 500 + 4000 = 3000$

[5] **Question 2:** Above what price would the number of tablets sold drop to 0?

$N(x) = 0 \Rightarrow -2x + 4000 = 0 \Rightarrow x = -\frac{4000}{-2} = 2000$

[5] **Question 3:** The amount of money made per day by HP, assuming they sell N tablets, is $M(x) = xN(x)$. Write the function $M(x)$ explicitly in terms of x only.

$M(x) = x(-2x + 4000) = -2x^2 + 4000x$

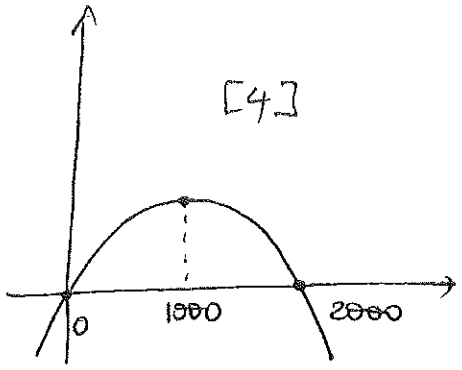
[2] **Question 4:** What is the name of this type of function? quadratic

[8] **Question 5:** Fully factor $M(x)$ and draw its signs table. Use the table to sketch $M(x)$.

$M(x) = -2x(x - 2000)$

		0		2000	
-2	$-$		$-$		$-$
x	$-$	0	$+$		$+$
$x-2000$	$-$		$-$	0	$+$
	$-$	0	$+$	0	$-$

[4]



[4]

[4] **Question 6:** What is price HP should set their tablets at to make as much money as possible? 1000

[4] **Question 7:** Above what price would they start losing money? 2000