MATH 3 2016, Midterm 2 version 1

Name: _____

Section : _____

Calculators are not allowed.

Read all the questions before you start working on any of them. Start with the ones you are most comfortable with, and continue with the other ones later. Always double-check your answers.

Relax, and do your best!

PROBLEM 1: SHORT QUESTIONS. [60 POINTS]. Give a short justification to each of your answers.

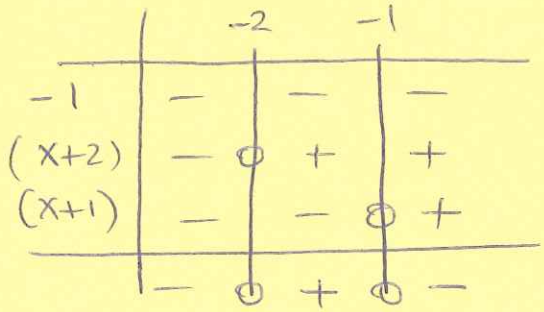
Given the function $f(x) = \frac{(x^2-4)(x+1)}{2-x}$

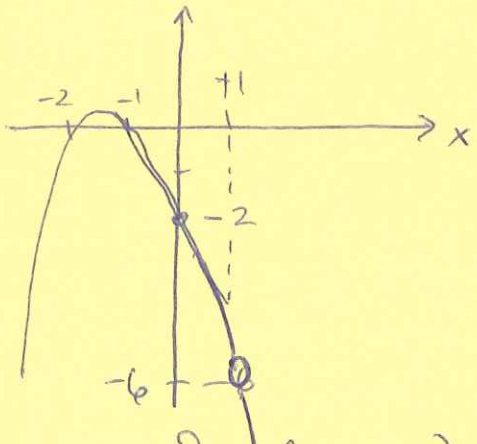
1. What is its domain of definition? ANSWER: $x \neq 2$ or $\mathbb{R} - \{2\}$ or $(-\infty, 2) \cup (2, +\infty)$

2. Simplify $f(x)$, for x inside the domain of definition. ANSWER: $f(x) = -(x+2)(x+1)$

$$f(x) = \frac{(x-2)(x+2)(x+1)}{2-x} = -(x+2)(x+1)$$

3. 4. Create a signs table for $f(x)$ (on the left) and sketch the function on the right, making sure to annotate your graph with any interesting point (intercepts, asymptotes, holes, etc..).





5. What is the domain of $\ln(f(x))$? ANSWER: $f(x) > 0 \Rightarrow \mathcal{D} = (-2, -1)$

6. Solve the inequality $\frac{2}{x-10} < 1$. ANSWER: $x \in$ _____

$\frac{2}{x-10} - 1 < 0$

$\frac{2-(x-10)}{x-10} < 0$

$\frac{2-x+10}{x-10} < 0$

$\frac{12-x}{x-10} < 0$

		10		12	
$12-x$	$+$	$+$	$\|$	0	$-$
$x-10$	$-$	0	$+$	$\|$	$+$
	$-$	$\|\|$	$+$	0	$-$

$\Rightarrow x \in (-\infty, 10) \cup (12, +\infty)$

Given the functions $f(x) = e^{2x}$ and $g(x) = \frac{1}{2}\ln(x)$

7. What is $f \circ g(x)$? $e^{2 \cdot \frac{1}{2}\ln x} = e^{\ln x} = x$ (Simplify if possible)

8. What is $g \circ f(x)$? $\frac{1}{2}\ln(e^{2x}) = \frac{1}{2} \cdot 2x = x$ (Simplify if possible)

9. TRUE or FALSE? : $g(x)$ is the inverse of $f(x)$. ANSWER: TRUE

10. TRUE or FALSE ?: $\frac{\log_2(3)}{\log_3(2)} = 1$. ANSWER: FALSE

11. Simplify $f(x) = \frac{2^{-x}4^{2x}}{8^x 4^{-x}}$. ANSWER: $f(x) =$ _____

$\frac{2^{-x}4^{2x}}{8^x 4^{-x}} = \frac{2^{-x}(2^2)^{2x}}{(2^3)^x (2^2)^{-x}} = \frac{2^{-x}2^{4x}}{2^{3x}2^{-2x}} = 2^{-x+4x-3x+2x}$

$= 2^{2x}$

Given the function $f(x) = 2e^x - 3$.

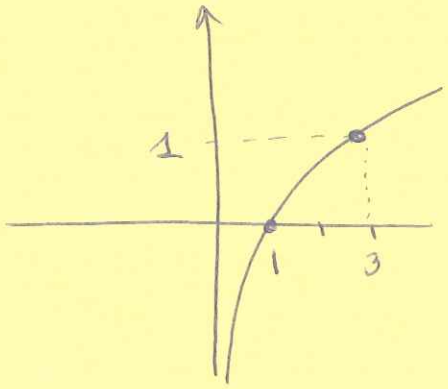
12. What is its y-intercept. ANSWER: $f(0) = 2e^0 - 3 = -1$

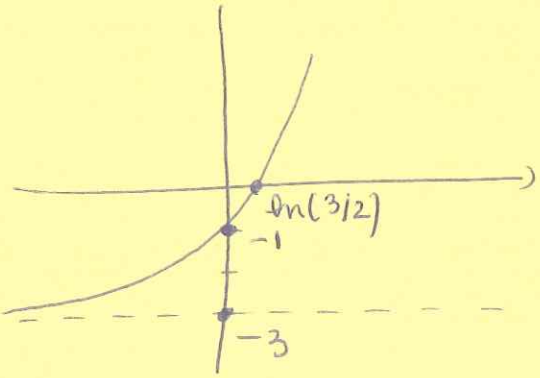
13. What is its x-intercept. ANSWER: $\ln(3/2)$

$f(x) = 0 \quad 2e^x - 3 = 0 \quad e^x = \frac{3}{2} \quad x = \ln(3/2)$

14. What is its horizontal asymptote. ANSWER: $y = -3$

15. 16. Sketch the functions $\log_3(x)$ and $f(x) = 2e^x - 3$, and annotate your graphs with all interesting points (intercepts, asymptotes, etc..)





17. Solve the equation $e^{2x+2} = 3^{-x}$. ANSWER:

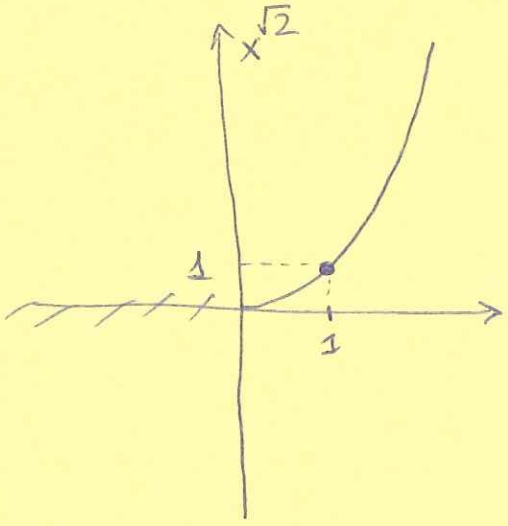
$e^{2x+2} = 3^{-x}$ $\quad \ln(e^{2x+2}) = \ln(3^{-x})$

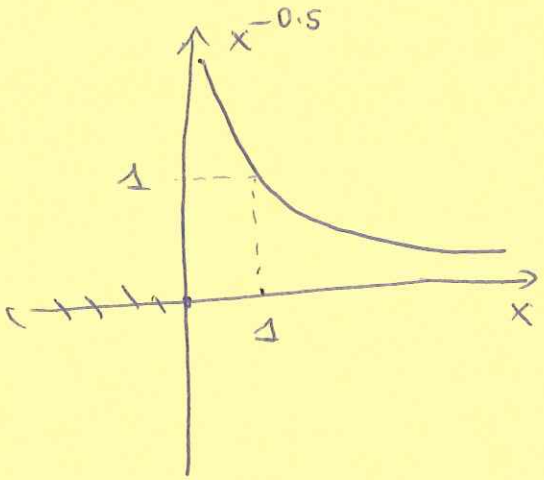
$2x+2 = -x\ln 3$ $\quad 2x + x\ln 3 = -2$

$x = \dfrac{-2}{2+\ln 3}$

18. TRUE OR FALSE ? $2^{x-1} = \frac{1}{2}e^{x\ln(2)}$: ANSWER: TRUE

19. 20. Sketch the functions $f(x) = x^{\sqrt{2}}$ and $g(x) = x^{-0.5}$. Make sure to mark the asymptotes and/or intercepts, as appropriate.





PROBLEM 2: RATIONAL FUNCTIONS.[20 POINTS] Consider the function $f(x) = \frac{2x+1}{2x-3}$

(a) What is the $x-$intercept? $-1/2$

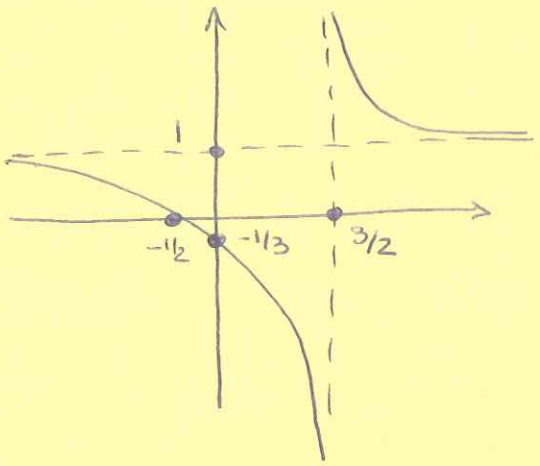
(b) What is the $y-$intercept? $-1/3$

(c) What is the vertical asymptote? $x = 3/2$

(d) What is the horizontal asymptote? $y = 1$

(e) Draw a signs table for $f(x)$ on the left, and using all the information you have to sketch $f(x)$ on the right. Include all intercepts, and asymptotes.

		$-1/2$		$3/2$	
$2x+1$	$-$	0	$+$		$+$
$2x-3$	$-$		$-$	0	$+$
	$+$	0	$-$	$\nexists$	$+$



(f) Calculate the inverse of $f(x)$.

$y = \frac{2x+1}{2x-3}$ $\quad (2x-3)y = 2x+1$ $\quad 2xy - 3y = 2x+1$

$2x(y-1) = 3y+1$ $\quad x = \frac{3y+1}{2(y-1)} = f^{-1}(y)$

$$f^{-1}(x) = \frac{3x+1}{2(x-1)}$$

(g) Verify that $f\left[f^{-1}(x)\right] = x$.

$$f(f^{-1}(x)) = \frac{2f^{-1}+1}{2f^{-1}-3} = \frac{2\left(\frac{3x+1}{2(x-1)}\right)+1}{2\left(\frac{3x+1}{2(x-1)}\right)-3} = \frac{\frac{3x+1+(x-1)}{x-1}}{\frac{3x+1-3(x-1)}{x-1}}$$

$$= \frac{3x+1+x-1}{3x+1-3x+3} = \frac{4x}{4} = x .$$

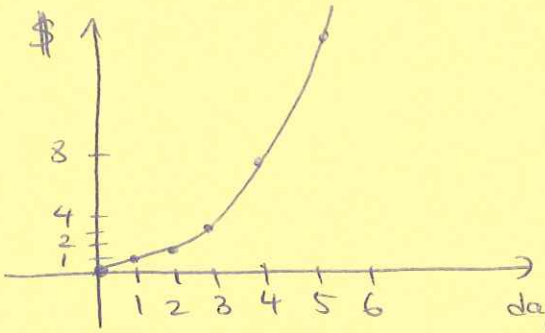
PROBLEM 3: APPLIED PROBLEM [20 POINTS]

A poor homeless mathematician who rescued the child of a rich multi-milionnaire is asked to pick a reward. The mathematician says "To give you time to prepare the reward, I'll come back every day for one month (30 days) to collect a little bit of money. The first day I will collect only 1 cent. The second day I will collect 2 cents, the third day I will collect 4 cents, and so forth. Every day, I will collect twice the amount of the previous day, for one month only. Do you agree to this reward?". The rich person, who had never heard of exponentials, agreed.

1. How many cents will the mathematician be picking up
   - on day 4? 8.
   - on day 6? 32.

2. How many cents will the mathematician be picking up after n days? Circle the correct function $r(n)$ (Reward r as a function of number of days n) among this list.
   - $r(n) = 2^n$
   - $r(n) = 2^{n+1}$
   - $r(n) = 2^{n-1}$
   - $r(n) = 2^{-n}$

3. Sketch the function you just found, making sure to annotate your graph with all the information you have gathered so far.



4. Express the function $r(n)$ as a natural exponential. $r(n) = e^{(n-1)\ln 2} = \frac{1}{2}e^{n\ln 2}$

5. After how many days can the mathematician expect to collect \$1,000,000 (which is equal to 10^8 cents)? (Hint: you may use the approximations $\ln(10) \simeq 2.1$, $\ln(2) \simeq 0.7$).

$2^{n-1} = 10^8$

$\ln(2^{n-1}) = \ln(10^8)$

$(n-1)\ln 2 = 8\ln 10$

$n-1 = \frac{8\ln 10}{\ln 2}$ $\quad n = \frac{8\ln 10}{\ln 2} + 1 \simeq 25$