

Algebra Workshop 4: Expressions with exponents

1 Rules for expressions with exponents

Review the following rules:

- $a^m a^n = a^{m+n}$ and $a^m a^{-n} = a^{m-n}$
- $(a^m)^n = a^{mn} = (a^n)^m$
- $a^m / a^n = a^{m-n}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$
- $a^{-m} = \frac{1}{a^m}$
- $a^0 = 1$
- $\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$

NOTE: These rules are applicable whether one has integer exponents, rational or irrational exponents.

PRACTICE: Each group gives one example of application for each of these 8 rules. Try to mix and match which “direction” you illustrate the rule. Be creative!

IMPORTANT OTHER RULES:

- $\sqrt{-5} = (-5)^{1/2}$ is not a real number - and the same applies for any even root of a negative number! But on the other hand $\sqrt[3]{-5} = (-5)^{1/3}$ exists and is equal to $-\sqrt[3]{5}$. This is true for any odd root of a negative number.
- $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$
- $\sqrt{x^2} = |x|$ and not $\sqrt{x^2} = x$. The same is true for all even root.
- $\sqrt[3]{x^3} = x$. The same is true for all odd roots.

PRACTICE: TRUE OR FALSE?

- $\sqrt{49} = -7$
- $\sqrt[3]{-27} = -3$
- $(\sqrt[3]{10})^3 = 10$
- $\sqrt{x+y} = \sqrt{x} + \sqrt{y}$
- $\sqrt{xy} = \sqrt{x}\sqrt{y}$
- $\sqrt{x^2 - y^2} = x - y$

- $\frac{\sqrt{x}}{\sqrt{y}} = \sqrt{\frac{x}{y}}$
- $\sqrt[3]{p^3 + q^3} = \sqrt[3]{p^3} + \sqrt[3]{q^3}$
- $\sqrt{10 + 6} = \sqrt{10} + \sqrt{6}$
- $\sqrt{10 + 6} = 4$
- $\sqrt{a^2} - a = 0$
- $\sqrt[5]{b^5} - b = 0$

2 Simplifying simple expressions

- $(1 + x^2)^2(x^2 + 1)^7 =$
- $\frac{3^x}{3^{1-x}} =$
- $(2x^2y^3z)^2 =$
- $\left(\frac{a}{b^{1/2}}\right)^{-1} =$
- $\left(\frac{x^3y^2z}{xy^2z^{-3}}\right)^{-3} =$
- $2^n 8^{n+1} =$
- $(2x^{1/2}y^{1/4}z)^2 =$
- $\left(\frac{a}{a^{1/2}}\right)^2 =$
- $\left(\frac{x^{3/2}y^{1/2}z}{xy^{3/2}z^{-3/2}}\right)^4 =$
- $4^{1/n} 8^{\frac{1}{2n}} =$

3 Simplifying harder expressions

- Simplify $(-1 - 2x - x^2)^3(x^2 + 2x + 1)^{-3} =$
- Simplify $\frac{\sqrt{1+x^2}}{(1+x^2)}$
- Simplify $\frac{(1-x^2)^{5/2}}{(1-x^2)^{3/2}}$
- Simplify $\frac{2^{2x}-1}{2^x+1}$
- Simplify $(3^{x/2} + 3^{-x/2})(3^{x/2} - 3^{-x/2})$