MATH 3: Practice for Midterm 1, Fall 2015

Name: _____

Calculators are not allowed.

Quickly read all the questions before you start working on any of them. Start with the ones you are most comfortable with, and continue with the other ones later. Always double-check your answers.

Relax, and do your best!

PROBLEM 1: SHORT QUESTIONS [60 POINTS] In the following questions, you are merely asked to provide the answer. No justification is needed. You should not be spending more than a 2 minutes per question. Each question is worth 3 points.

1. What is the equation of the line passing through the points $(1, 1)$ and $(-2, 2)$?

2. Find the linear function $f(x)$ such that $f(0) = 1$ and $f(3) = 2$.

Given the functions $f(x) = \frac{1}{x}$ and $g(x) = \sqrt{1-x}$

3. Write down, and then simplify the expression $f(x) - f(x-1)$.

4. What is the domain of $f(x)$? _____

5. What is the domain of $g(x)$? _____

6. What is the domain of $\frac{f}{g}(x)$? _____

7.,8. Sketch the function $f(x) = -(x+1)^3 + 1$ and $g(x) = \frac{1}{(x-3)^2} - 4$

Given the function $f(x) = -x^2 - 2x + 3$ and its graph:

9. Complete the square

10. What are the coordinates of the vertex? _____

11. Does it open up or down? _____

12. What is the equation of the tangent at the $y-$intercept? _____

13. Does the function have roots? If so, what are they?

14. Based on this information, sketch the parabola $y = -x^2 - 2x + 3$, making sure to annotate your graph correctly.

15. Factor the quadratic function $f(x) = -x^2 + 2x + 5$

16. Factor the following expression by grouping, and make sure your result is *fully* factored:
$f(x) = 2x^4 - 4x^3 - x^2 + 2x$.

Given the function $f(x) = \frac{3x^3-3x}{x^2-4}$:

17. What is the name of this type of function? _____

18. What are the $x-$intercepts? _____

19. What are the vertical asymptotes? _____

20. What is the equation of the oblique asymptote? _____

PROBLEM 2. [20 POINTS] Consider the function $f(x) = -x^3 + 3x^2 - 2x$.

(a) Behavior for large x:

- What is $f(x)$ approximately equal to for large $|x|$? _____
- When x tends to $-\infty$, $f(x)$ goes to _____
- When x tends to $+\infty$, $f(x)$ goes to _____

(b) Factor the function: _____

(c) Determine the $x-$ and $y-$ intercepts

$x-$intercept(s): _____ $y-$intercept: _____

(d) Draw a signs table (make sure to include the zeros and infinity signs as appropriate)

(f) Sketch the function

PROBLEM 3. APPLIED PROBLEM [20 POINTS].

This problem guides you through a mathematical proof that the largest possible rectangular area of perimeter 1 (one) is achieved when that rectangular area is actually a square.

Question 1: Considering a rectangle of length x and width y. How does y relate to x given the constraints of the problem?

Question 2: What is the area of the rectangle as a function of x and y? Using the results of Question 1, what is the area of the rectangle as a function of x only?

Question 3: What is the name of this type of function? _____

Question 4: Using the standard methods we have learned, sketch this function (Hint: find the $x-$ and $y-$intercepts, etc..

Question 5: Using the standard methods we have learned, find the position of the maximum of this function.

Question 6: Why does this mean the rectangle is a square?