# MATH 3 formula sheet

All formula with an asterisk must be known by heart. All others are optional and will be given to you in exams if required.

## 1 Lines, circles and points

The equation of a line with slope $s$ and y-intercept $b$ is

$$y = f(x) = sx + b \qquad (*) \tag{1}$$

If the line goes through 2 points A and B with coordinates $(x_A, y_A)$ and $(x_B, y_B)$ then

$$s = \frac{y_B - y_A}{x_B - x_A} \qquad (*) \tag{2}$$

The distance between two points $A(x_A, y_A)$ and $B(x_B, y_B)$ is

$$d = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2} \qquad (*) \tag{3}$$

The equation of a circle of radius $R$ centered on $A(x_A, y_A)$ is

$$(x - x_A)^2 + (y - y_A)^2 = R^2 \qquad (*) \tag{4}$$

## 2 Quadratic equations

$$y = f(x) = ax^2 + bx + c \qquad (*) \tag{5}$$

The graph of $y = f(x)$ is a parabola. It has a minimum (i.e. parabola opens upwards) if $a > 0$. It has a maximum (i.e. parabola opens downwards) if $a < 0$. The minimum/maximum is at the location $x_m$ with

$$x_m = -\frac{b}{2a} \qquad (*) \tag{6}$$

It has roots (i.e it intersects the $x$-axis) when $y = f(x) = 0$. The solutions to this equation depends on the value of $D$:

$$D = b^2 - 4ac \qquad (*) \tag{7}$$

- if $D < 0$ there are no solutions. The parabola does not intercept the $x$-axis.
  The function $f(x) = ax^2 + bx + c$ cannot be factored.
- if $D = 0$ there is one solution. The parabola just touches the $x$-axis at the point

  $$x_1 = x_m = -\frac{b}{2a} \qquad (*) \tag{8}$$

  The function $f(x) = ax^2 + bx + c$ is factored as

  $$f(x) = a(x - x_1)^2 \qquad (*) \tag{9}$$

- if $D > 0$ there are two solutions. The parabola intercepts the $x$-axis in the two points

$$x_1 = \frac{-b - \sqrt{D}}{2a} \; , \; x_2 = \frac{-b + \sqrt{D}}{2a} \qquad (*) \tag{10}$$

  The function $f(x) = ax^2 + bx + c$ is factored as

$$f(x) = a(x - x_1)(x - x_2) \qquad (*) \tag{11}$$

## 3 Polynomial functions

$$y = f(x) = a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n \qquad (*) \tag{12}$$

$a_n$ is the leading coefficient, $n$ is the order of the polynomial.

The factored form of $f$ is

$$f(x) = a_n(x - x_1)(x - x_2)(x - x_3) \ldots (x - x_m) q(x) \qquad (*) \tag{13}$$

where $x_i$ are all possible solutions to $f(x) = 0$ and $q(x)$ is a polynomial of order $n - m$, leading coefficient 1, with no roots ($q(x) \neq 0$).

## 4 Rational functions

$$y = f(x) = \frac{p(x)}{q(x)} \qquad (*) \tag{14}$$

where $p(x)$ and $q(x)$ are polynomial functions.

The roots of $p(x)$ are the roots of $f(x)$. The roots of $q(x)$ are the asymptotes of $f(x)$.

## 5 Power functions

$$y = f(x) = x^a \qquad (*) \tag{15}$$

Properties:

$$x^{a+b} = x^a x^b \qquad (*) \tag{16}$$

$$x^{-a} = \frac{1}{x^a} \qquad (*) \tag{17}$$

$$x^{a-b} = \frac{x^a}{x^b} \qquad (*) \tag{18}$$

$$x^{ab} = (x^a)^b = (x^b)^a \qquad (*) \tag{19}$$

## 6 Exponential functions

Exponential in base $a$:

$$y = f(x) = a^x \text{ with } a > 0 \qquad (*) \tag{20}$$

Natural exponential (exponential in base $e$ with $e = 2.71828\ldots$):

$$y = f(x) = e^x = \exp(x) \qquad (*) \tag{21}$$

Properties of all exponential functions:

$$a^{x+z} = a^x a^z \qquad (*) \tag{22}$$

$$a^{-x} = \frac{1}{a^x} \qquad (*) \tag{23}$$

$$a^{x-z} = \frac{a^x}{a^z} \qquad (*) \tag{24}$$

$$a^{xz} = (a^x)^z = (a^z)^x \qquad (*) \tag{25}$$

## 7 Logarithmic functions

Logarithm in base $a$ is the inverse of the exponential in base $a$:

$$y = \log_a(x) \text{ is equivalent to } x = a^y \qquad (*) \tag{26}$$

Natural logarithm (logarithm in base $e$) is the inverse of the natural logarithm:

$$y = \log_e(x) = \ln(x) \text{ is equivalent to } x = e^y \qquad (*) \tag{27}$$

Inverse relations:

$$\log_a(a^x) = x \qquad (*) \tag{28}$$

$$a^{\log_a(x)} = x \qquad (*) \tag{29}$$

$$\ln(e^x) = x \qquad (*) \tag{30}$$

$$e^{\ln(x)} = x \qquad (*) \tag{31}$$

Properties of all logarithmic functions (where $a$ is a positive constant).

$$\log_a(xy) = \log_a(x) + \log_a(y) \qquad (*) \tag{32}$$

$$\log_a\left(\frac{1}{x}\right) = -\log_a(x) \qquad (*) \tag{33}$$

$$\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y) \qquad (*) \tag{34}$$

$$\log_a(x^c) = c\log_a(x) \qquad (*) \tag{35}$$

Relations for changing bases:

- From an exponential function in base $a$ to the natural exponential:

$$a^x = e^{x\ln a} \qquad (*) \tag{36}$$

- From a logarithmic function in base $a$ to the natural logarithm:

$$\log_a(x) = \frac{\ln x}{\ln a} \qquad (*) \tag{37}$$

## 8 Trigonometric functions

The basic trigonometric functions are:

$$y = f(x) = \sin(x) \qquad (*) \tag{38}$$
$$y = f(x) = \cos(x) \qquad (*) \tag{39}$$
$$y = f(x) = \tan(x) = \frac{\sin(x)}{\cos(x)} \qquad (*) \tag{40}$$
$$y = f(x) = \cot(x) = \frac{\cos(x)}{\sin(x)} \qquad (*) \tag{41}$$
$$y = f(x) = \sec(x) = \frac{1}{\cos(x)} \qquad (*) \tag{42}$$
$$y = f(x) = \csc(x) = \frac{1}{\sin(x)} \qquad (*) \tag{43}$$
$$\tag{44}$$

Table of values you have you know (*):

| Angle (degree) | Angle (radian) | sin | cos | tan |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0 |
| 30 | $\pi/6$ | 0.5 | $\sqrt{3}/2$ | $1/\sqrt{3}$ |
| 45 | $\pi/4$ | $\sqrt{2}/2$ | $\sqrt{2}/2$ | 1 |
| 60 | $\pi/3$ | $\sqrt{3}/2$ | 0.5 | $\sqrt{3}$ |
| 90 | $\pi/2$ | 1 | 0 | not defined |

Properties:

$$\cos^2 x + \sin^2 x = 1 \qquad (*) \tag{45}$$
$$\sin(2x) = 2\sin x \cos x \qquad (*) \tag{46}$$
$$\cos(2x) = \cos^2 x - \sin^2 x \qquad (*) \tag{47}$$
$$\tan(2x) = \frac{2\tan x}{1 - \tan^2 x} \tag{48}$$

Other addition/multiplication formula

$$\cos(a+b) = \cos a \cos b - \sin a \sin b \tag{49}$$
$$\cos(a-b) = \cos a \cos b + \sin a \sin b \tag{50}$$
$$\sin(a+b) = \sin a \cos b + \cos a \sin b \tag{51}$$
$$\sin(a-b) = \sin a \cos b - \cos a \sin b \tag{52}$$
$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b} \tag{53}$$
$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b} \tag{54}$$