### 2.2.10 Graphing quadratics

Let's now recap everything we know about the graphs of quadratic functions based on their mathematical expression. Given $f(x) = ax^2 + bx + c$,

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- Given $D = b^2 - 4ac$:

Based on all this information (much of which is actually redundant) we can easily graph the parabola.

EXAMPLE 1: $f(x) = 2x^2 - 3x + 1$

EXAMPLE 2: $f(x) = -2x^2 - 8x - 8$

EXAMPLE 3: $f(x) = -x^2 + x - 6$

### 2.2.11 Minimization and maximization

One of the very reasons Calculus was invented was to answer questions of optimization in real world problems, i.e. given a function, what are its minima and maxima. In general, answering this question requires Calculus tools – something you will learn in the following courses. However, for quadratics, we can do this quite easily since the minimum/maximum of the function coincides with the vertex of the parabola – a point whose coordinates we know.

EXAMPLE 1: OPTIMIZING REVENUE FOR A PRODUCT. (See Example 1 Chapter 2.6 of the textbook)

A market analysis for Texas Instrument has collected data suggesting that the average number of calculators sold in any given month depends on their price, in the following way:

How many calculators can they expect to sell if the price if \$20, and how much money will they make?

How many calculators can they expect to sell if the price if \$50, and how much money will they make?

How many calculators can they expect to sell if the price if \$100, and how much money will they make?

As we can see,

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This suggests that there is an "optimal" price that will guarantee the largest return! To find that price, let's model the total return as a function of the selling price $p$.

This is a quadratic function! Let's graph that function using everything we know.

We therefore see that the function has a maximum at

Example 2: Rocket science

When an object is thrown upward, its height as a function of time depends on its initial velocity $v_0$, as well as the gravity of the Earth $g$, as

How does the highest point the object can reach depend on $g$ and $v_0$?

### 2.2.12 A couple of interesting things about quadratics.

Axis of symmetry: The vertical line that passes through the vertex of the parabola is an axis of symmetry for the parabola. This implies, in particular, that

We can verify that this is the case in the examples we saw earlier:

Sign of a quadratic: As we saw at the beginning of this Section on quadratics, for large enough $x$ (both positive and negative)

Since $x^2$ is always positive, this means that, for large enough $x$

With this in mind, we can now figure out what the sign of $f(x)$ is for all values of $x$, for all three possible scenarios. Indeed, the only way $f(x)$ can change sign is when the parabola actually crosses the $x-$axis, that is, when $f(x)$ has two different roots.

This is quite useful when trying to find, for instance, the domain of definition of functions defined as the square roots of quadratics.

- $f(x) = \sqrt{x^2 - x + 6}$

- $f(x) = \sqrt{-x^2 - x + 20}$