

6.7.8 Applications of trigonometric formula: the beating phenomenon

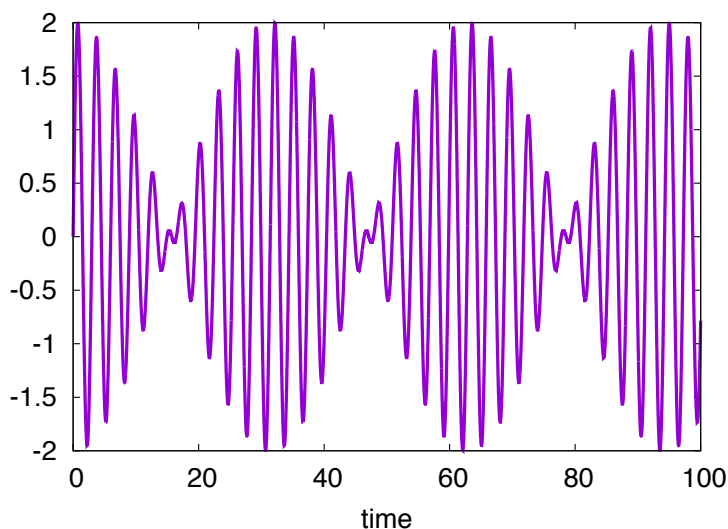
Watch video on sound beating: <https://www.youtube.com/watch?v=IYeV2Wq82fw>

As we see in the video, two musical notes played at the same time but with slightly different tones interfere with one another and produce a phenomenon called *beating*. To understand beating, note that sound is actually a wave, i.e. an oscillation of the air between the instrument and our eardrum. Different sounds are simply oscillations that have different periods. The equation that describes one sound wave is

Note that we often think of the properties of sound in terms of the frequency of the sound wave, rather than its period. The frequency and the period are related by

Low frequency sounds are low-pitched sounds. High frequency sounds are high-pitched sounds. When an instrument plays together two notes of different pitch, but similar amplitude then the equation for the sum of the two waves is simply:

Suppose now, as suggested in the video, that the two waves have very similar frequencies (equivalently, very similar periods). For instance, let's pick $f_1 = 2$ and $f_2 = 2.2$, and graph the resulting sum of two waves:



This shows that the sum of two notes of nearly the same frequency, and with the same amplitude, results in a modulation of the amplitude of the sound : this is the beating phenomenon!

To understand the phenomenon, remember the formula derived in the previous lecture relating the product of two sines to the sum of two sines:

As it turns out, we can use this formula to prove another formula:

Using this equation we can then see that

This shows that the *sum* of the two waves is *also* equal to the product of two waves, one whose frequency is the average of the two original ones, and one whose frequency is half the difference between the two original ones. Now, if the two initial frequencies are nearly-identical, then the average frequency $(f_1 + f_2)/2$ is nearly the same as f_1 or f_2 , while the frequency difference $(f_2 - f_1)/2$ is very small. This low frequency is called the *beat frequency*.

When a low frequency signal multiplies a high-frequency one, the resulting function looks like the high-frequency oscillation, but instead of having a constant amplitude, the amplitude varies in time according to the low-frequency signal. This is exactly what we are seeing here.

6.8 Solving trigonometric equations

Textbook Section 6.3

Trigonometric equations are equations that involve trigonometric functions, and that need to be solved for the unknown variable. The tricky thing about trigonometric equations is that sometimes they do not have solutions, but when they do have solutions, they often have infinitely many of them – and all of them need to be found. Let's work by examples to see what may happen.

EXAMPLE OF EQUATIONS THAT DO NOT HAVE SOLUTIONS.

- $\cos(4x + 2) = 5$.

- $\cos^2 a + \sin^2 a = 2$.

- $\cos^2 y + 2 \sin^2 y = 3$.

EXAMPLES OF BASIC TRIGONOMETRIC EQUATIONS THAT HAVE INFINITELY MANY SOLUTIONS

- Solve the equation $\cos(x) = \frac{1}{2}$

- Solve the equation $\sin(3x) = \frac{\sqrt{3}}{2}$

- Solve the equation $3 \cos(x) = 1$

- Solve the equation $\cos^2(x) + \cos(x) - 2 = 0$

EXAMPLES OF MORE ADVANCED TRIGONOMETRIC EQUATIONS THAT REQUIRE USING SOME TRIGONOMETRIC FORMULAS

- Solve the equation $\sin(2x) = \cos(x)$

- Solve the equation $3 \cos(a) + 3 = 2 \sin^2(a)$

EXAMPLES OF TRIGONOMETRIC FUNCTIONS THAT CAN BE SOLVED GRAPHICALLY (OR NEARLY GRAPHICALLY)

- Solve the equation $\sin(a) = \cos(a)$

- Solve the equation $\sin(a) = 2 \cos(a)$