6.7 Trigonometric formulas

Textbook Section 6.4

There are a few very important formulas in trigonometry, which you will need to know as a preparation for Calculus. These formulas are very useful when trying to simplify complicated expressions which involve trigonometric functions.

6.7.1 The basic formulas

We've already seen a few of these formulas: the three basic ones are

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EXAMPLES OF USE:

- Simplify: $1 + \tan^2(x)$

- Simplify: $1 - \frac{\cot^2(x)}{\csc^2(x)}$

- Prove that $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$

The list of trigonometric identities which can be proved using the basic formulas is endless. See Textbook Examples 1-8 for instance.

6.7.2 The addition formulas

The addition formulas relate the sines and cosines of *sums* of angles to the *products* of sines and cosines of basic angles. The 4 formulas are

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Examples: These examples show that these formulas indeed work:

- $\cos(\pi/3 - \pi/6)$
- $\cos(\pi/3 + \pi/3)$
- $\sin(\pi/4 + \pi/4)$

Also, they confirm our hunch that $\sin(x)$ and $\cos(x)$ are the "same" function but displaced by $\pi/2$. Indeed, we saw from the graph that $\sin(x) = \cos(x - \pi/2)$. We can now prove this mathematically:

NOTE: There is no equivalent *product* formulas: there is no simple identity for $\cos(ab)$ and $\sin(ab)$. On the other hand, the addition formulas can be used backward to prove the following identities:

Altogether these identities can be used to prove another nearly-infinite number of identities.

EXAMPLE 1: Prove that

$$\frac{\sin(a-b)}{\cos(a)\cos(b)} + \frac{\sin(b-c)}{\cos(b)\cos(c)} + \frac{\sin(c-a)}{\cos(c)\cos(a)} = 0$$

EXAMPLE 2: Prove that $\cos^2 a - \sin^2 b = \cos(a-b)\cos(a+b)$

EXAMPLE 3: Prove that $\sin^2 a - \sin^2 b = \sin(a-b)\sin(a+b)$

6.7.3 The double-angle formulas

As a consequence of the addition formulas, we have 3 more formulas which are called the *double-angle* formulas because they express the sine and cosine of the angle $2a$ in terms of the sine and cosine of the angle a. These formulas are

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To show them, note that

Again, these double-angle formula can be used to simplify trigonometric expressions.

EXAMPLE 1: Simplify $\frac{\sin(2x)}{\sin(x)} - \frac{\cos(2x)}{\cos(x)} = \sec x$

EXAMPLE 2: Show that $\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$

EXAMPLE 3: Show that $\cos(2x) + 1 = 2\cos^2(x)$

6.7.4 Summary

Of the various formulas above, only a few have to be known by heart:

- The definition of the various functions:

- The Pytagorean identity:

- The symmetry property:

- The periodicity property:

- The phase-shift property:

- The 2 double-angle formulas:

All of the other formulas can be recovered from combining these ones – so don't clog your memory with formulas... In what follows, we will now use these formulas to solve equations involving trigonometric functions.