6.6 The inverse trigonometric functions

Textbook Section 6.1

6.6.1 Definitions

We define the three basic inverse trigonometric functions:

Horizontal line test? However, note how neither of the three basic trigonometric functions pass the horizontal line test. As a consequence, the domain of definition of the inverse functions is limited to a region where the function does pass the test.

Based on that, we can now graph the $\sin(x)$ function and its inverse:

As well as the $\cos(x)$ function and its inverse:

And finally, the $\tan(x)$ function and its inverse:

As for any function and their inverse, $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ so, for any x within the domain of definition of the respective functions, we have:

Examples:

- What is $\arcsin(\sin(\pi/3))$?
- What is $\sin((\arcsin(2))$?
- What is $\arctan(\tan(\pi/3))$?
- What is $\tan((\arctan(2))$?

Finally, note that in Calculus, the expressions $\sin(\cos^{-1}(x))$ and $\cos(\sin^{-1}(x))$ will often come up. These cannot be simplified as easily as the other ones, but on the other hand, by combining them with the Pythagorean formula, we can still get a nicer formula:

6.6.2 Applications of inverse trigonometric functions

EXAMPLE 1: A contractor is installing a door in a narrow corridor. The door is 3 ft long, centered in the wall, and the corridor is 4 feet wide (see diagram). What is the maximum angle this door could open?

EXAMPLE 2: The water level of tides in Santa Cruz in Steamer's lane is given by $h(t) = 15+6\cos\left(2+\frac{\pi}{6}t\right)$. At what time is the low tide?